

Definition and Realization of Terrestrial Reference Systems: Application to ITRS and ETRS89

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OUTLINE

- **Definition of a TRS**
- **TRS Realization by a Frame (TRF)**
- **International Terrestrial Reference System (ITRS) and its realization: the International Terrestrial Reference Frame (ITRF)**
- **ITRF Geodetic & Geophysical Results**
- **European Terrestrial Reference System (ETRS89)**

Ideal Terrestrial Reference System

A tridimensional reference frame (mathematical sense)
Defined in an Euclidian affine space of dimension 3:

Affine Frame (O,E) where:

O: point in space (**Origin**)

E: vector base: orthogonal with the same length:

- vectors co-linear to the base (**Orientation**)
- unit of length (**Scale**)

$$\lambda = \|\vec{E}_i\|_{i=1,2,3}$$

$$\vec{E}_i \cdot \vec{E}_j = \lambda^2 \delta_{ij}$$

$$(\delta_{ij} = 1, \quad i = j)$$

Ideal Terrestrial Reference System in the Context of Space Geodesy

- **Origin:** Geocentric: Earth Center of Mass
- **Scale:** SI Unit
- **Orientation:** Equatorial (Z axis is the direction of the Earth pole)

Transformation between TRSs

$$X_2 = X_1 + T + DX_1 + R.X_1 + \underbrace{O_2}_{\cong 0}$$

X, T, R & D are function of time:

$$\dot{X}_2 = \dot{X}_1 + \dot{T} + \underbrace{\dot{D}\dot{X}_1}_{\approx 0} + \dot{D}X_1 + \underbrace{\dot{R}\dot{X}_1}_{\approx 0} + \dot{R}X_1$$

$$\dot{T} = \begin{pmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{pmatrix}, \quad \dot{R} = \begin{pmatrix} 0 & -\dot{R}_3 & \dot{R}_2 \\ \dot{R}_3 & 0 & -\dot{R}_1 \\ -\dot{R}_2 & \dot{R}_1 & 0 \end{pmatrix}$$

==> 14 degrees of freedom to realize a TRS

Crust-based TRF

The instantaneous position of a point on Earth Crust at epoch t could be written as :

$$X(t) = X_0 + \dot{X} \cdot (t - t_0) + \sum_i \Delta X_i(t)$$

- X_0 : point position at a reference epoch t_0
- \dot{X} : point linear velocity
- $\Delta X_i(t)$: high frequency time variations:
- solid Earth tide
 - ocean loading
 - atmospheric loading
 - geocenter motion
 - ...

Implementation of a TRF

- **Definition at a chosen epoch, by selecting 7 datum parameters, tending to satisfy the theoretical definition of the corresponding TRS**
- **A law of time evolution, by selecting 7 rates of the 7 datum parameters, **assuming linear motion for both stations and frame parameters!****
- **==> 14 parameters are needed to define a TRF**

How to define the 14 parameters ?

« Datum definition »

- **Origin & rate: CoM (Dynamical Techniques)**
 - **Scale & rate: depends on physical parameters**
 - **Orientation: conventional**
 - **Orient. Rate: conventional: Geophysical meaning (Tectonic Plate Motion)**
-
- **==> Lack of information for some parameters:**
 - **Orientation & rate (all techniques)**
 - **Origin & rate in case of VLBI**
 - **==> Rank Deficiency in terms of Normal Eq. System**

Implementation of a TRF in practice

The Normal Equation is under the form:

$$N.(\Delta X) = K$$

where $\Delta X = X - X_{apr}$ are the linearized unknowns

Singular Normal Equation: rank deficiency equal to the # of parameters not reduced by the observations

Additional constraints are necessary :

- **Tight constraints** ($\sigma \leq 10^{-10}$) m
 - **Removable constraints** ($\sigma \cong 10^{-5}$) m
 - **Loose constraints** ($\sigma \geq 1$) m
 - **Minimum constraints** (applied over datum parameters)
- } applied over coord.

Datum Definition using minimum constraints (1/2)

The starting point is the standard relation:

$$X_2 = X_1 + A\theta$$

$$X_i = (x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i)^T$$

$$\theta = (T1, T2, T3, D, R1, R2, R3, \dot{T}1, \dot{T}2, \dot{T}3, \dot{D}, \dot{R}1, \dot{R}2, \dot{R}3)^T$$

Least Squares equations give for θ :

$$\theta = \overbrace{(A^T A)^{-1} A^T}^{\mathbf{B}} (X_2 - X_1)$$

A : design matrix of partial derivatives

The equation of minimum constraints is under the form:

$$B(X_2 - X_1) = 0 \quad (\Sigma_\theta)$$

Normal Eq.

$$B^T \Sigma_\theta^{-1} B (X_2 - X_1) = 0$$

$$A = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 & & & & & & \\ 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 & & 0 & & & & \\ 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 & & & & & & \\ & & & & & & & 1 & 0 & 0 & x_i^0 & 0 & z_i^0 & -y_i^0 \\ & & & \approx 0 & & & & 0 & 1 & 0 & y_i^0 & -z_i^0 & 0 & x_i^0 \\ & & & & & & & 0 & 0 & 1 & z_i^0 & y_i^0 & -x_i^0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Datum Definition using minimum constraints (2/2)

Considering the Normal Equation :

$$N_{nc}(\Delta X) = K \quad (1)$$

where $\Delta X = X - X_{apr}$ are the linearized unknowns

Selecting a reference solution (X_R)

$$B^T \Sigma_{\theta}^{-1} B(\Delta X) = B^T \Sigma_{\theta}^{-1} B(X_R - X_{apr}) \quad (2)$$

Cumulating (1) and (2), we have:

$$(N_{nc} + B^T \Sigma_{\theta}^{-1} B)(\Delta X) = K + B^T \Sigma_{\theta}^{-1} B(X_R - X_{apr})$$

International Association of Geodesy

Associated Space Geodesy Services

- **International Earth Rotation and Reference Systems Service (IERS) (1988)**
- **Intern. GNSS Service (IGS) (1994)**
- **Intern. Laser Ranging Service (ILRS) (1998)**
- **Intern. VLBI Service (IVS) (1999)**
- **Intern. DORIS Service (IDS) (2003)**

<http://www.iag-aig.org/>

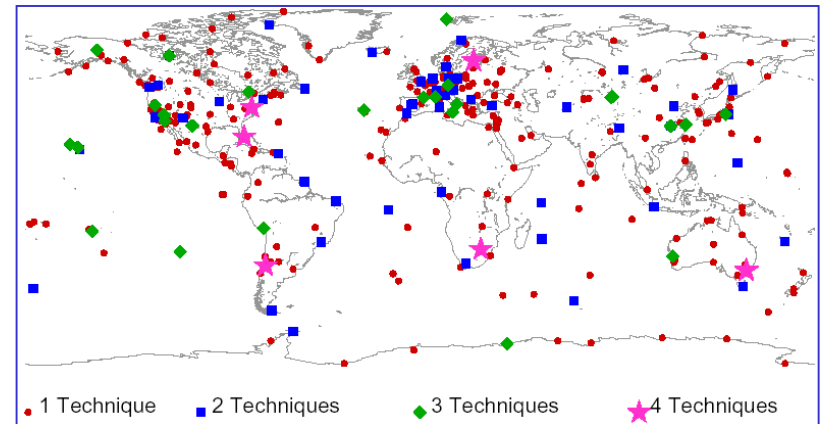
International Terrestrial Reference System (ITRS): Definition

- **Origin:** Center of mass of the whole Earth, including oceans and atmosphere
- **Unit of length:** meter SI, consistent with TCG (Geocentric Coordinate Time)
- **Orientation:** consistent with BIH (Bureau International de l'Heure) orientation at 1984.0.
- **Orientation time evolution:** ensured by using a No-Net-Rotation-Condition w.r.t. horizontal tectonic motions over the whole Earth

International Terrestrial Reference System (ITRS)

- Realized and maintained by **ITRS Product Center** of the IERS
- Its Realization is called International Terrestrial Reference Frame (**ITRF**)
- Set of station positions and velocities, estimated by combination of VLBI, LLR, SLR, GPS and DORIS individual TRF solutions
- **Based on Co-location sites**

Adopted by IUGG in 1991 for all Earth Science Applications



About 700 stations located on about 500 sites

Available: ITRF88, 89, ..., 2000

Latest: ITRF2005

<http://itrf.ensg.ign.fr/ITRF/>

Co-location Site

- Site where two or more space geodesy close instruments (hundred meters) are operating
- Precisely surveyed in three dimensions, using classical or GPS geodesy
- Differential coordinates (DX, DY, DZ) are available

$$DX_{(GPS,VLBI)} = X_{VLBI} - X_{GPS}$$



GPS

VLBI



SLR

DORIS



Combination in the era of times series

- **Time series of station positions allow to detect:**
 - **station non-linear and seasonal motions, discontinuities and other problems**
 - **geocenter motion**
 - **loading effects (common mode)**
 - **Ensure TRF & EOP consistency in the combination**

ITRF2005

- **For 1st time, use Time Series of Station Positions :**
 - **Daily (VLBI)**
 - **Weekly (GPS, SLR & DORIS)**
- **and Earth Orientation Parameters:**
 - Polar Motion (x_p, y_p)**
 - Universal Time (UT1) (Only from VLBI)**
 - Length of Day (LOD) (Only from VLBI)**

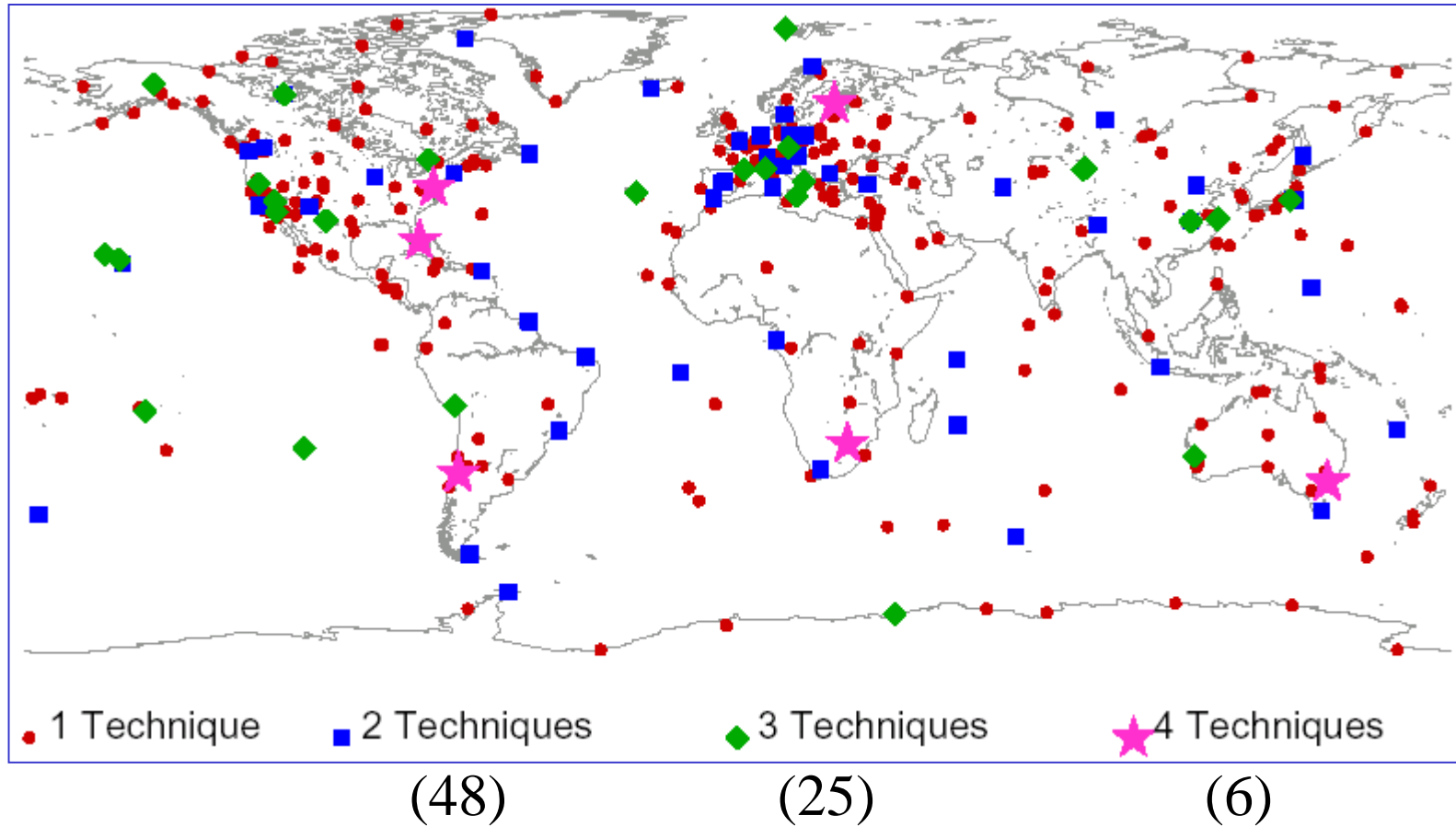
Submitted data

TC - AC	Time-span	Type of constraints/solution
IVS VLBI	1980.0-2006.0	Normal Equation
ILRS SLR	1992.9-2005.9	Loose; Var-Covar
IGS GPS	1996.0-2006.0	Minimal; Var-Covar
IDS - IGN DORIS	1993.0-2006.0	Loose; Var-Covar
IDS -LCA DORIS	1993.0-2005.8	Loose; Var-Covar

Daily

Weekly

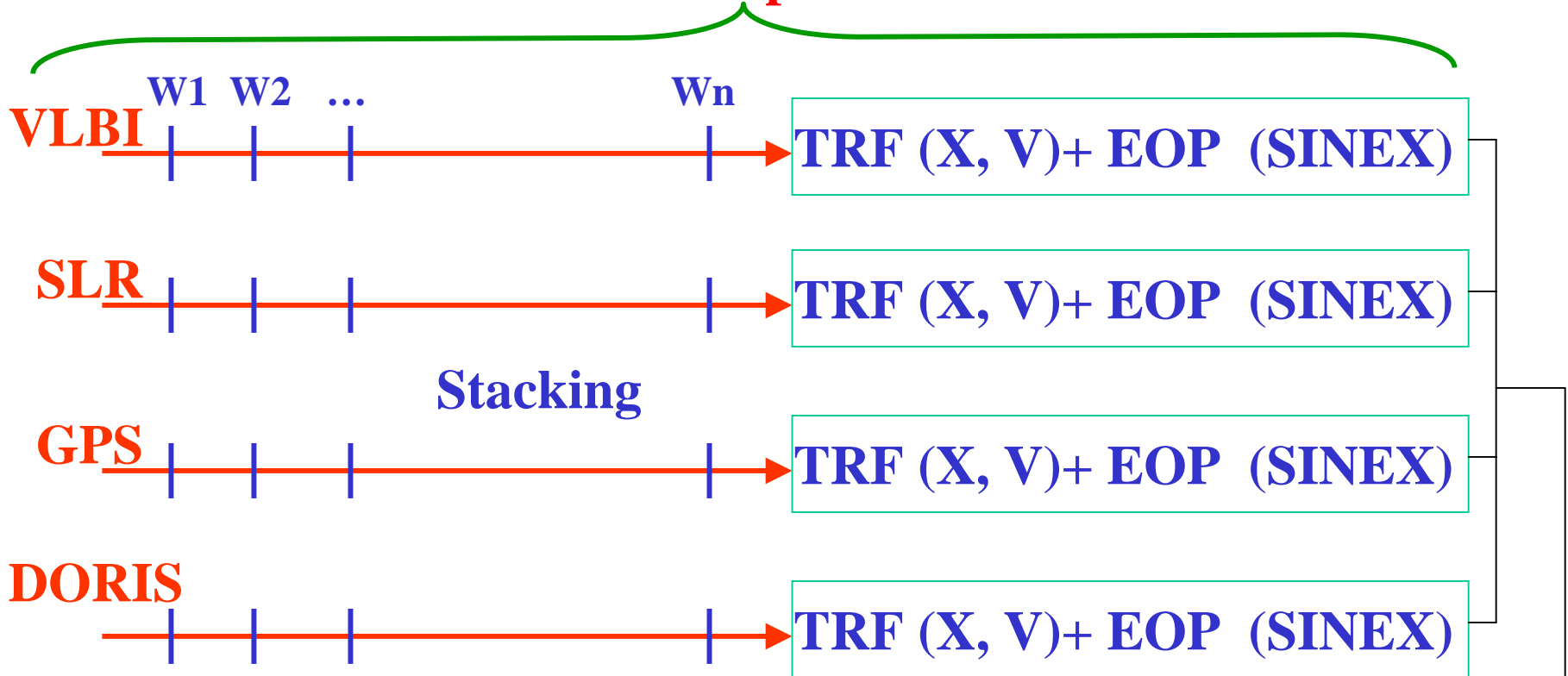
ITRF2005 Co-locations



175 tie vectors

ITRF2005 Derivation

Step 1



**Local Ties
100 SINEXs**

**Combination
ITRF2005**

TRF (X, V) + EOP (SINEX)

Step 2

Methods of implementation

- (1) Select an external solution (X_R) and apply minimum constraints equations:

$$X_R = X_c + A\theta \xrightarrow{\theta = 0} (A^T A)^{-1} A^T (X_R - X_c) = 0$$

Or

- (2) Considering linear transf. parameter P :

$$P(t) = P(t_0) + \dot{P} \cdot (t - t_0)$$

Intrinsic conditions:

$$P(t_0) = 0 \quad \& \quad \dot{P} = 0 \quad \text{or}$$

$$\begin{cases} \sum_{k \in K} P_k(t_k) = 0 \\ \sum_{k \in K} \frac{P_k(t_k)}{(t_k - t_0)^{-1}} = 0 \end{cases}$$

TRF & EOP time series Combination

CA_{TRF} Software

INPUT: $X(t)$, $\mathbf{EOP}(t)$ in daily/weekly/monthly SINEX files

OUTPUT: $X(t_0)$, \dot{X} , $\mathbf{EOP}(t)$, $(\underbrace{T_x, T_y, T_z}_{\text{Geocenter}}, D, R_x, R_y, R_z)$

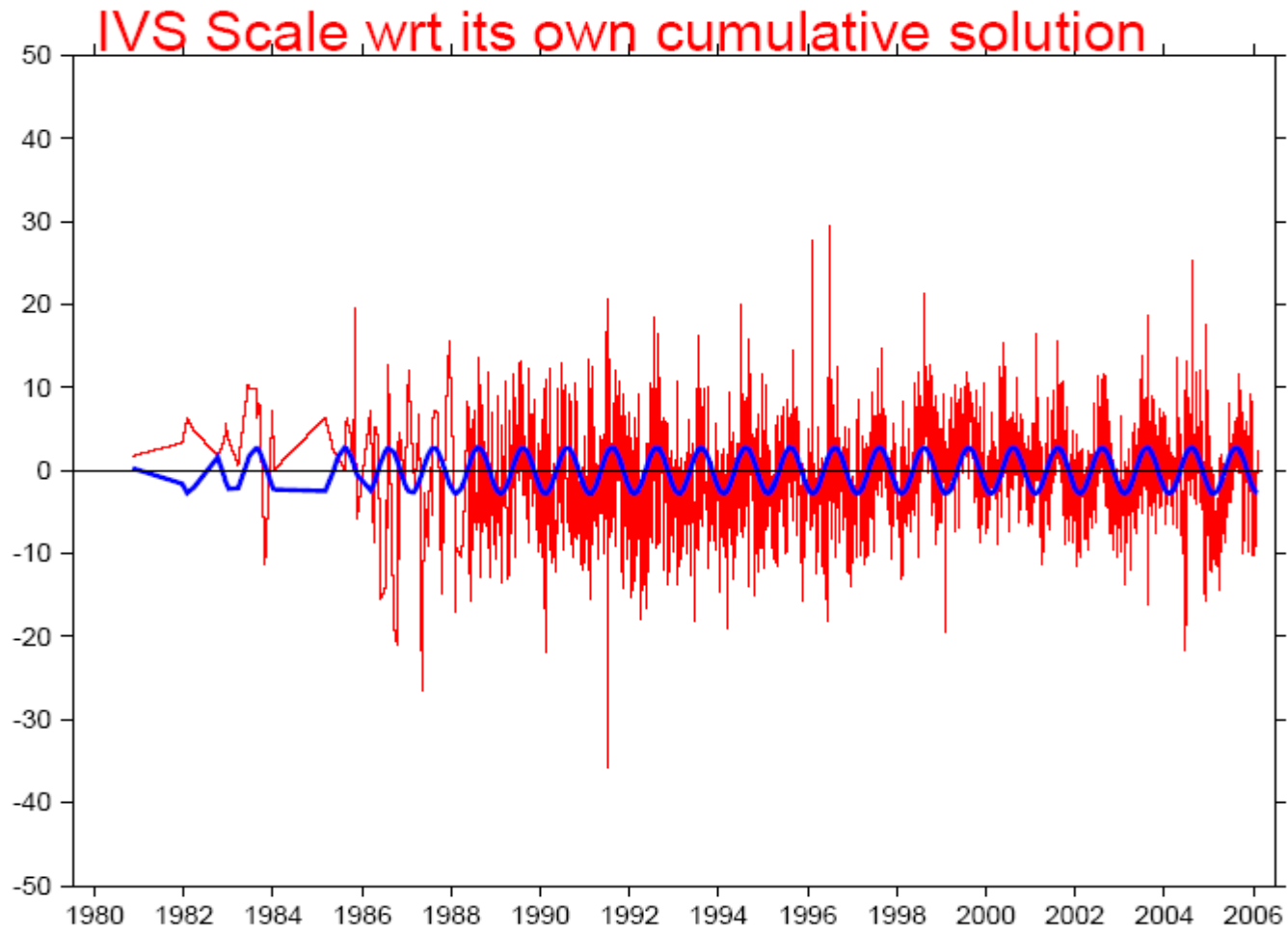
$$\begin{cases} X_s^i = X_{itr}^i + (t_s^i - t_0) \dot{X}_{itr}^i + T_k + D_k X_{itr}^i + R_k X_{itr}^i \\ \quad + (t_s^i - t_k) \left[\dot{T}_k + \dot{D}_k X_{itr}^i + \dot{R}_k X_{itr}^i \right] \\ \dot{X}_s^i = \dot{X}_{itr}^i + \dot{T}_k + \dot{D}_k X_{itr}^i + \dot{R}_k X_{itr}^i \end{cases}$$

**Datum Definition with
Minimum or Intrinsic
Constraints**

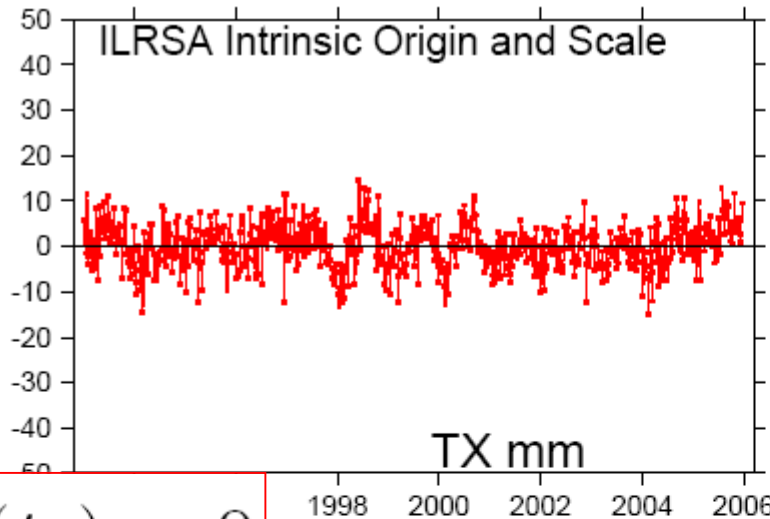
$$\begin{cases} x_s^p & = x^p + R2_k \\ y_s^p & = y^p + R1_k \\ UT_s & = UT - \frac{1}{f} R3_k \\ \dot{x}_s^p & = \dot{x}^p + \dot{R}2_k \\ \dot{y}_s^p & = \dot{y}^p + \dot{R}1_k \\ LOD_s & = LOD + \frac{\Lambda_0}{f} \dot{R}3_k \end{cases}$$

- Matching common EOP parameters at UT noon
- Propagate at UT noon if rates are available

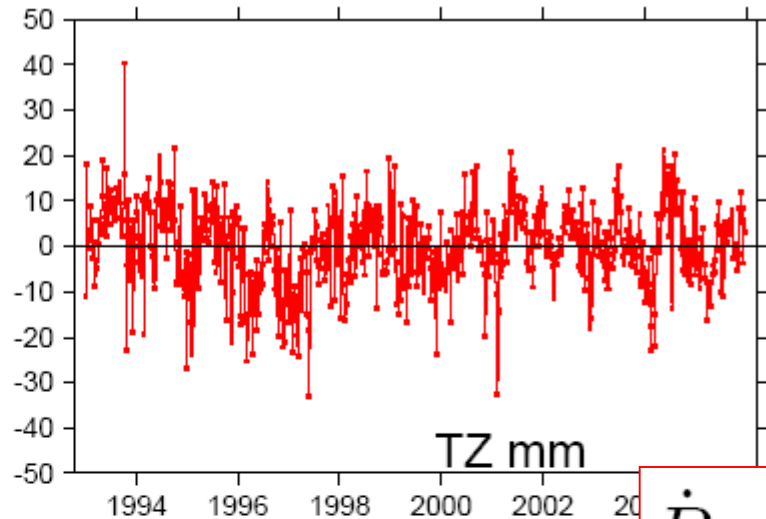
VLBI Scale Variation



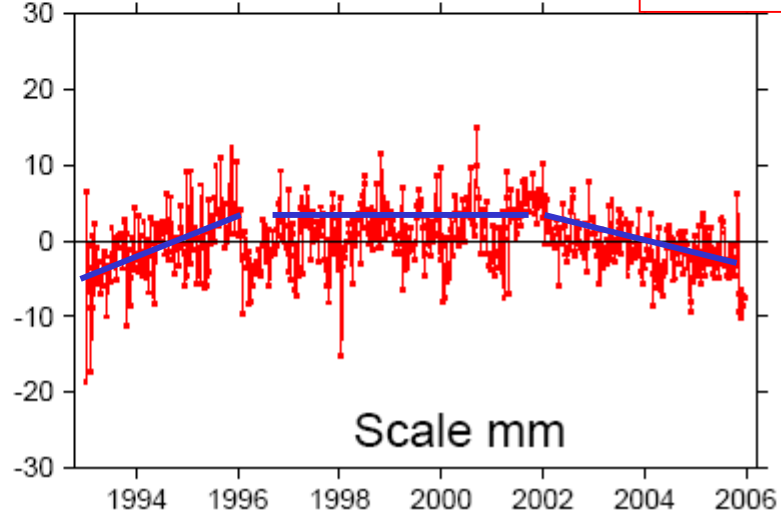
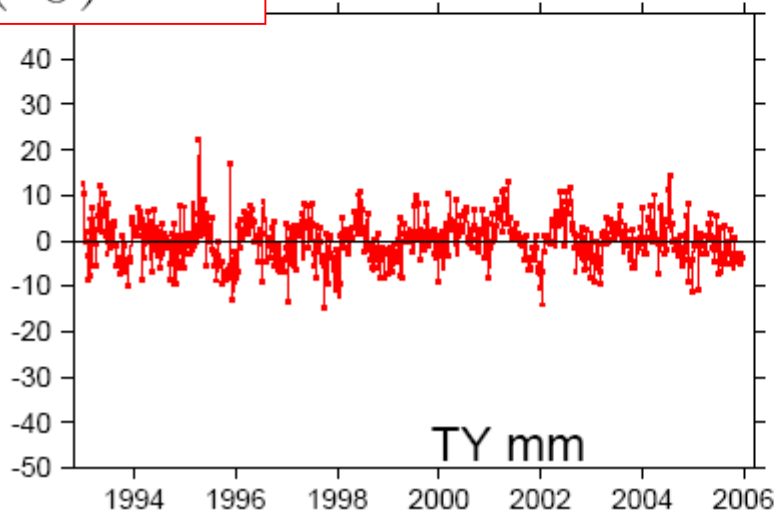
SLR Intrinsic Origin & Scale



$$P(t_0) = 0$$



$$\dot{P} = 0$$

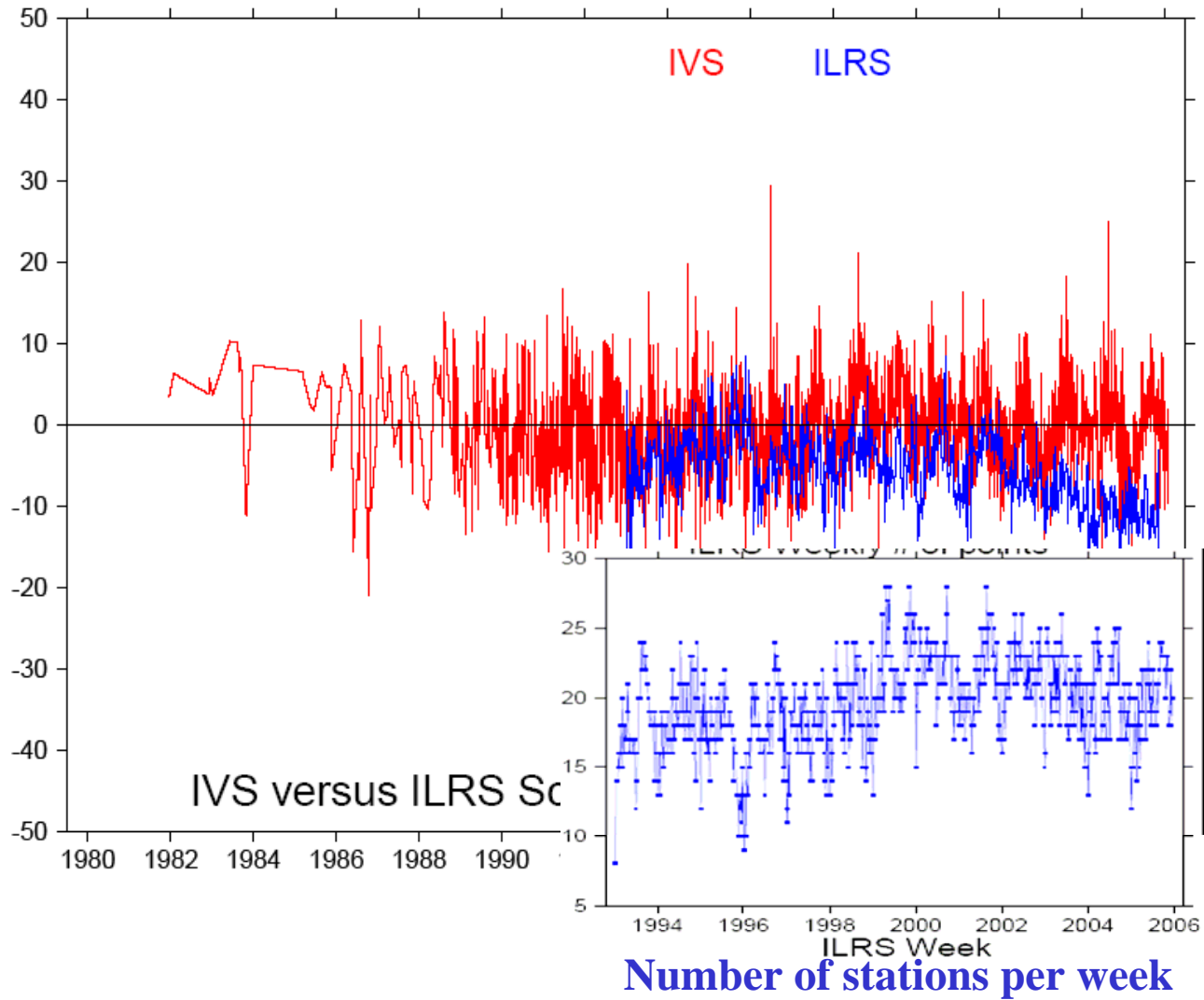


ITRF2005 Combination

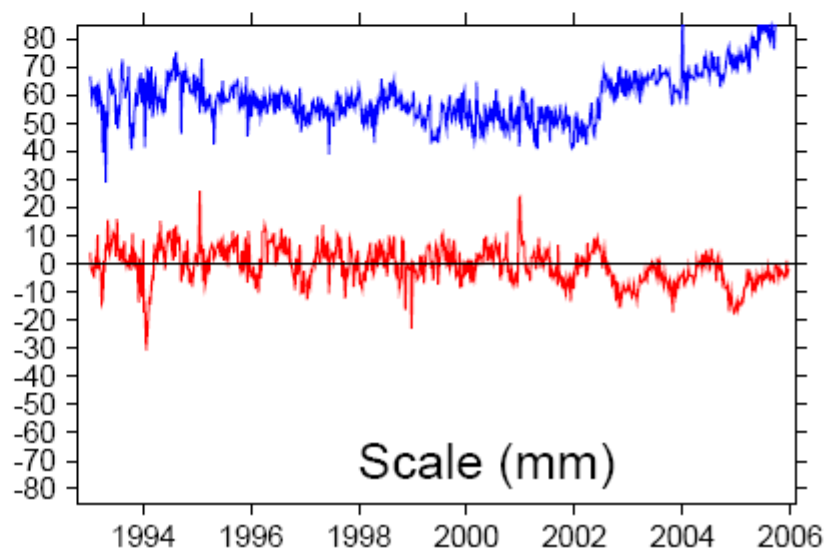
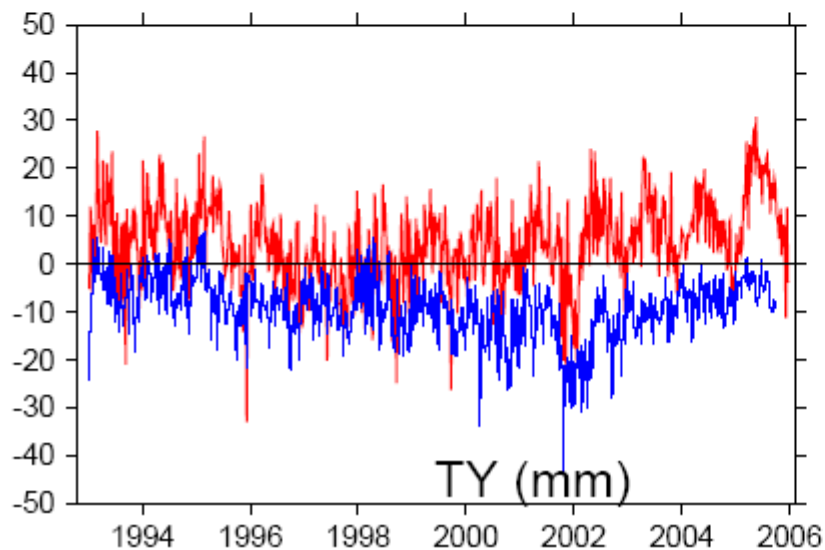
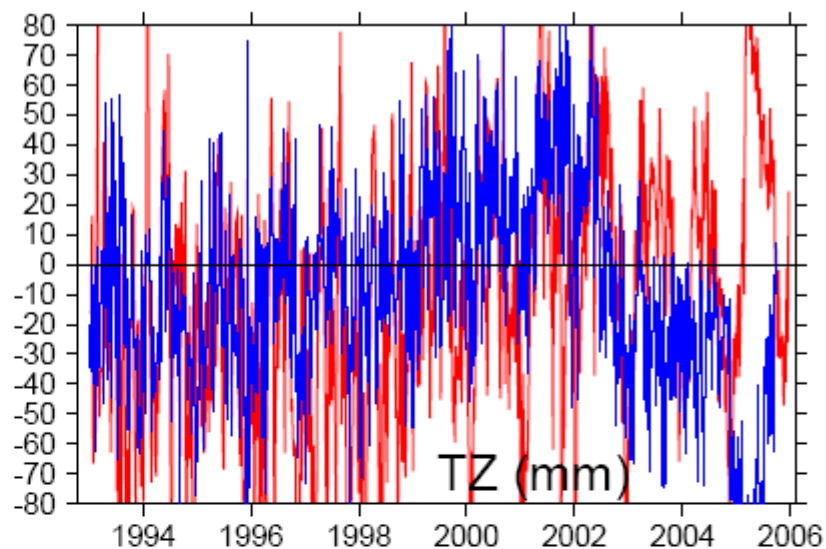
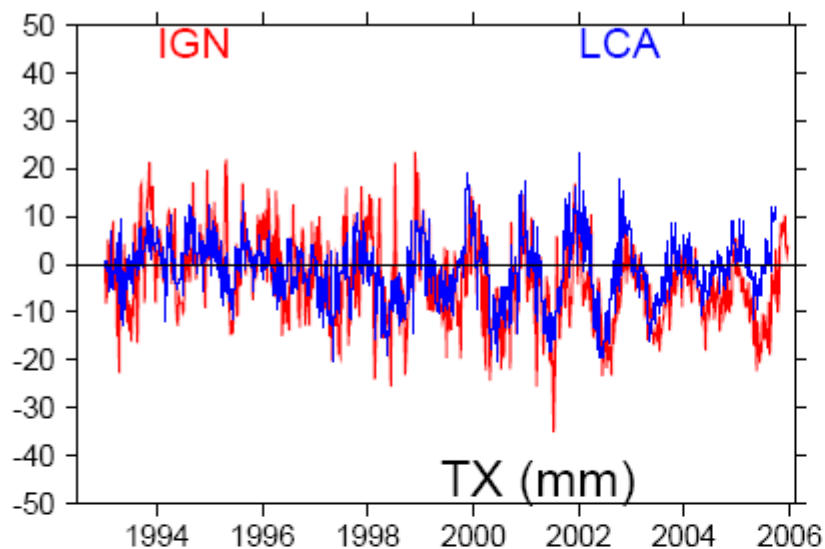
- Long-term solutions of the 4 techniques: positions, velocities & EOPs
- 100 Local Ties SINEX files where ~50 % with full variance covariance
- **Origin:** by fixing to zero the translations of the SLR solution
- **Scale:** by fixing to zero the scale of the VLBI solution
- **Orientation & NNR:** Aligement to ITRF2000

==> A scale bias of 1 ppb (6.3 mm) between VLBI & SLR solutions

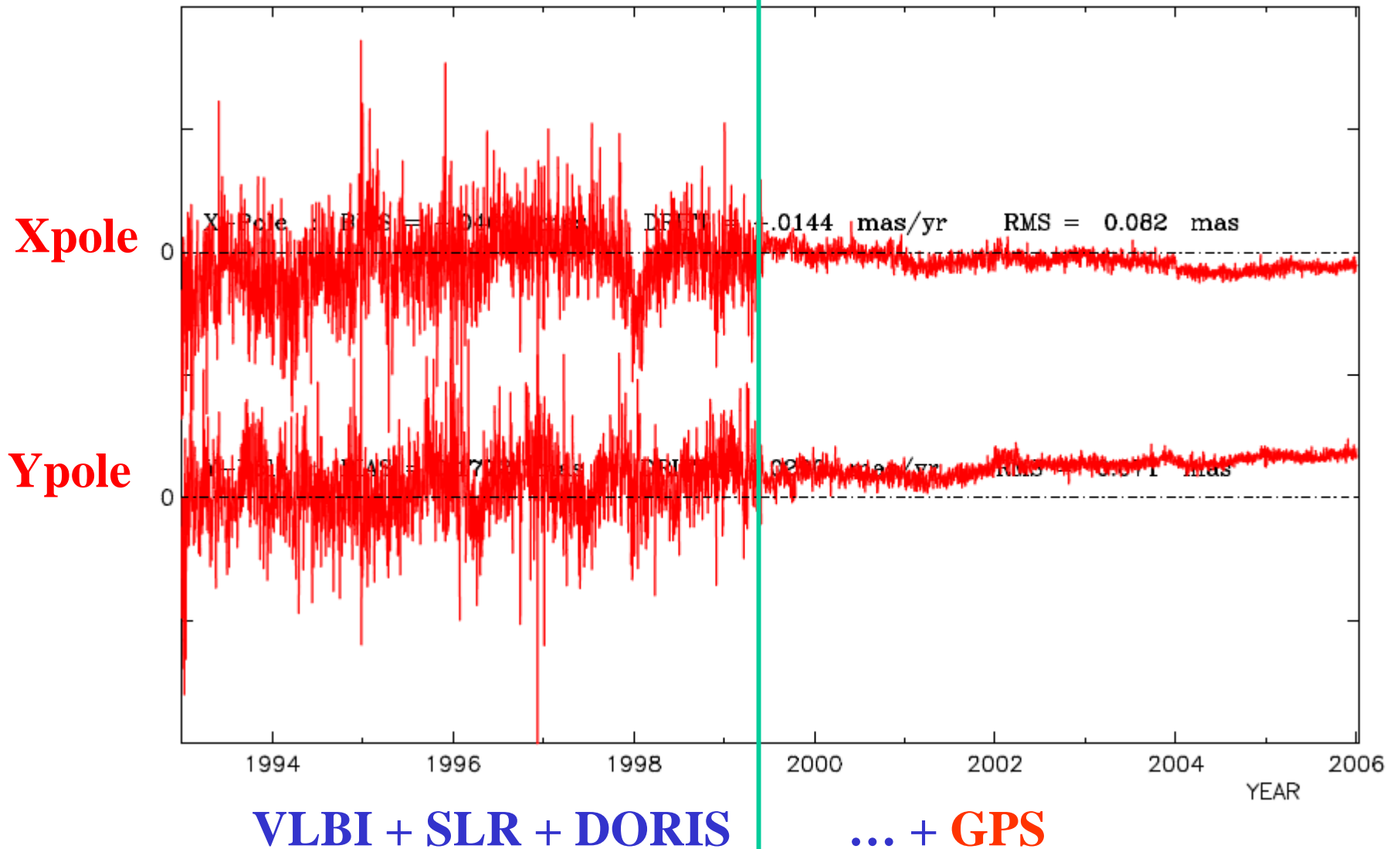
VLBI vs SLR Scale wrt ITRF2005



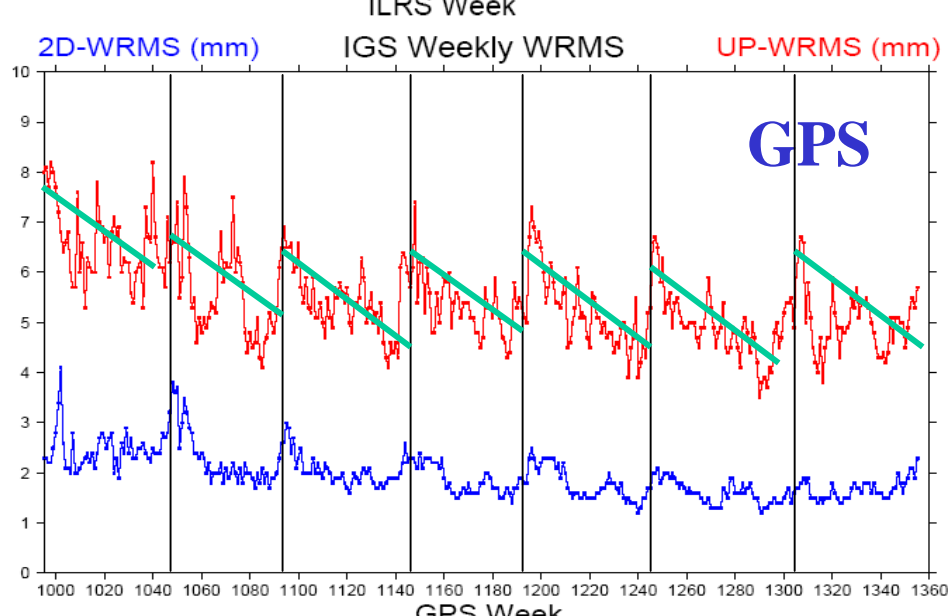
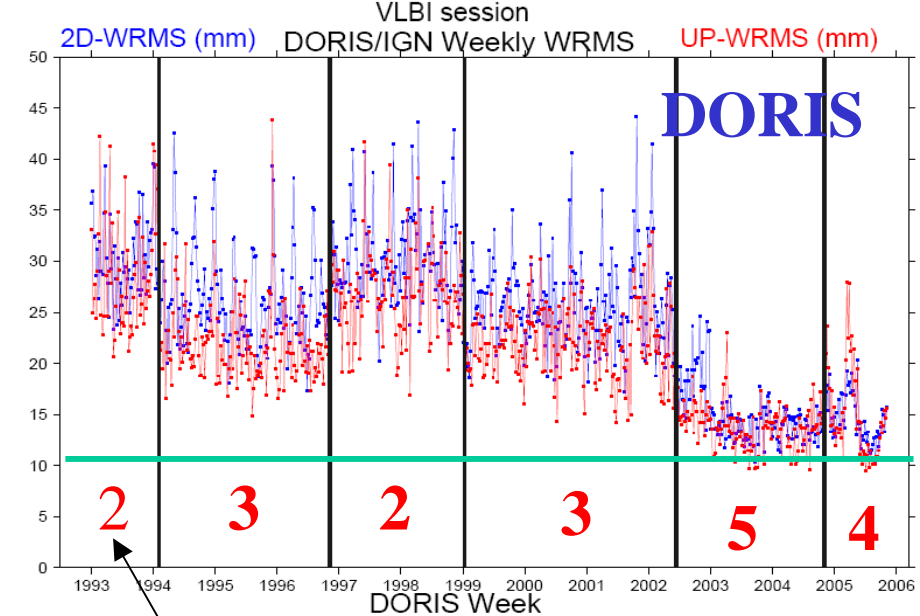
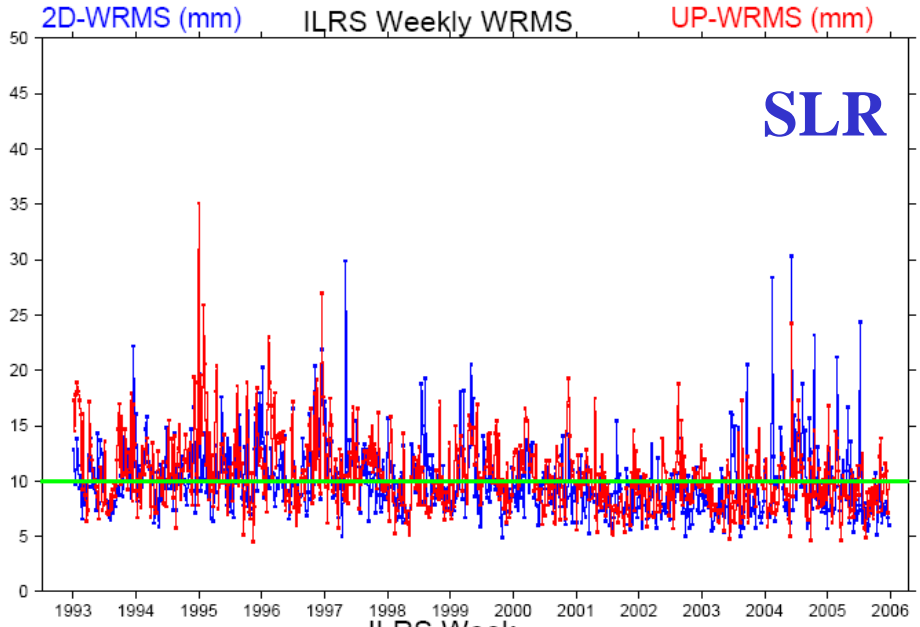
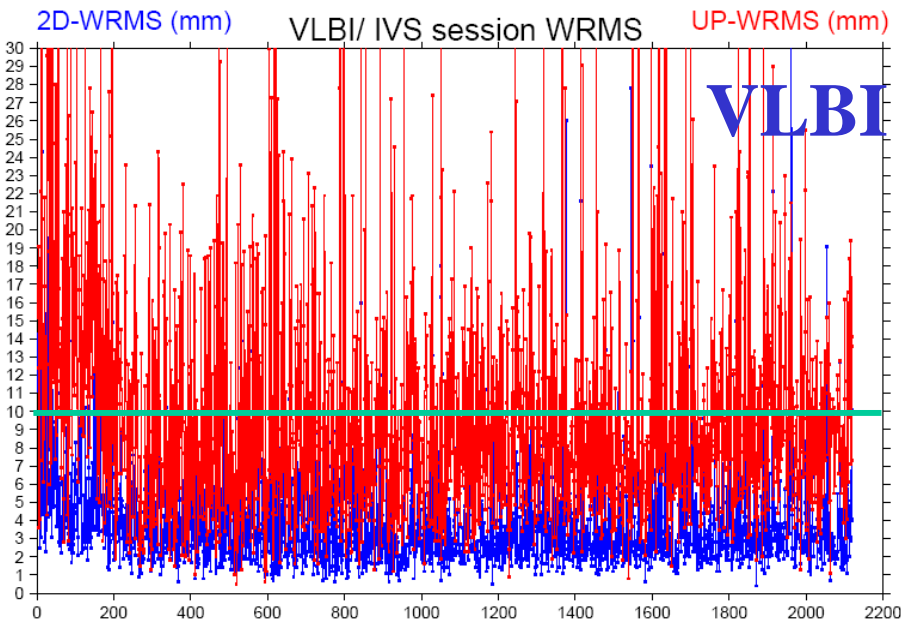
DORIS Origin and Scale DORIS variations wrt ITRF2005



Polar Motion differences ITRF2000(5) - IERS C04



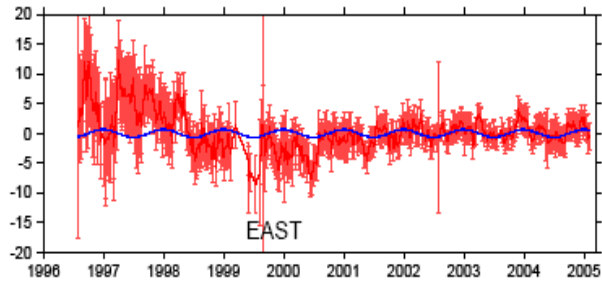
Positioning Performance from ITRF2005 Experience



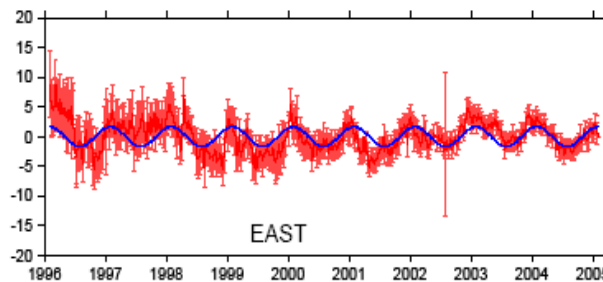
Number of satellites used

Seasonal Variations GPS/IGS Sites

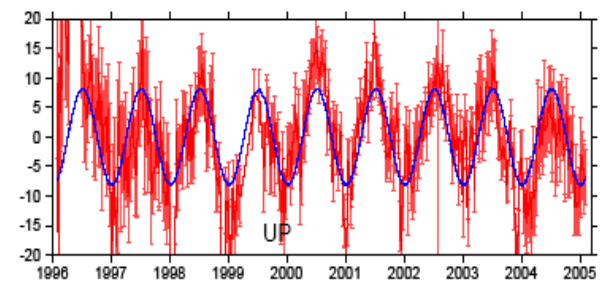
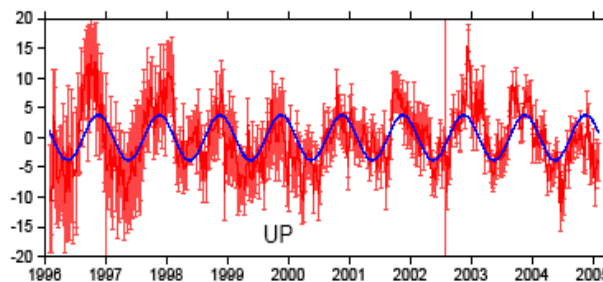
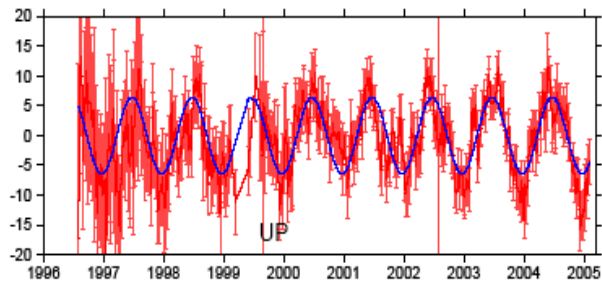
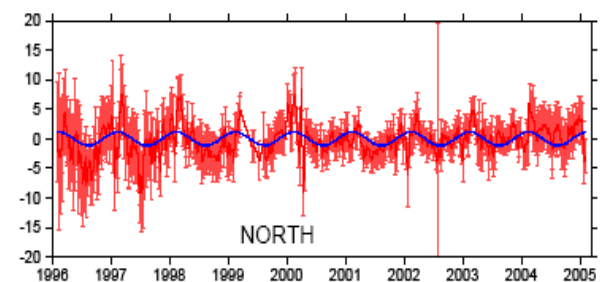
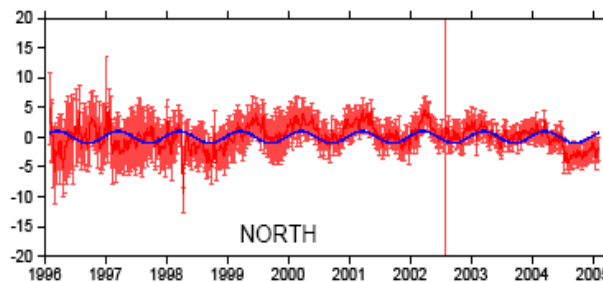
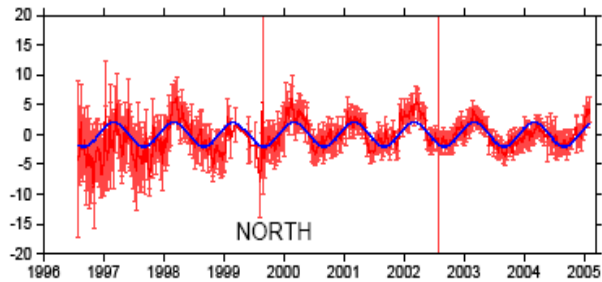
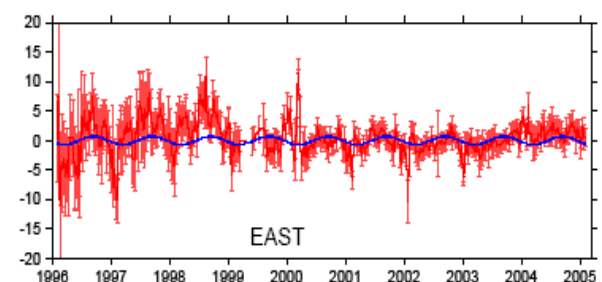
BAHR Annual Amplitude and Phase (mm)



DRAO Annual Amplitude and Phase (mm)



IRKT Annual Amplitude and Phase (mm)



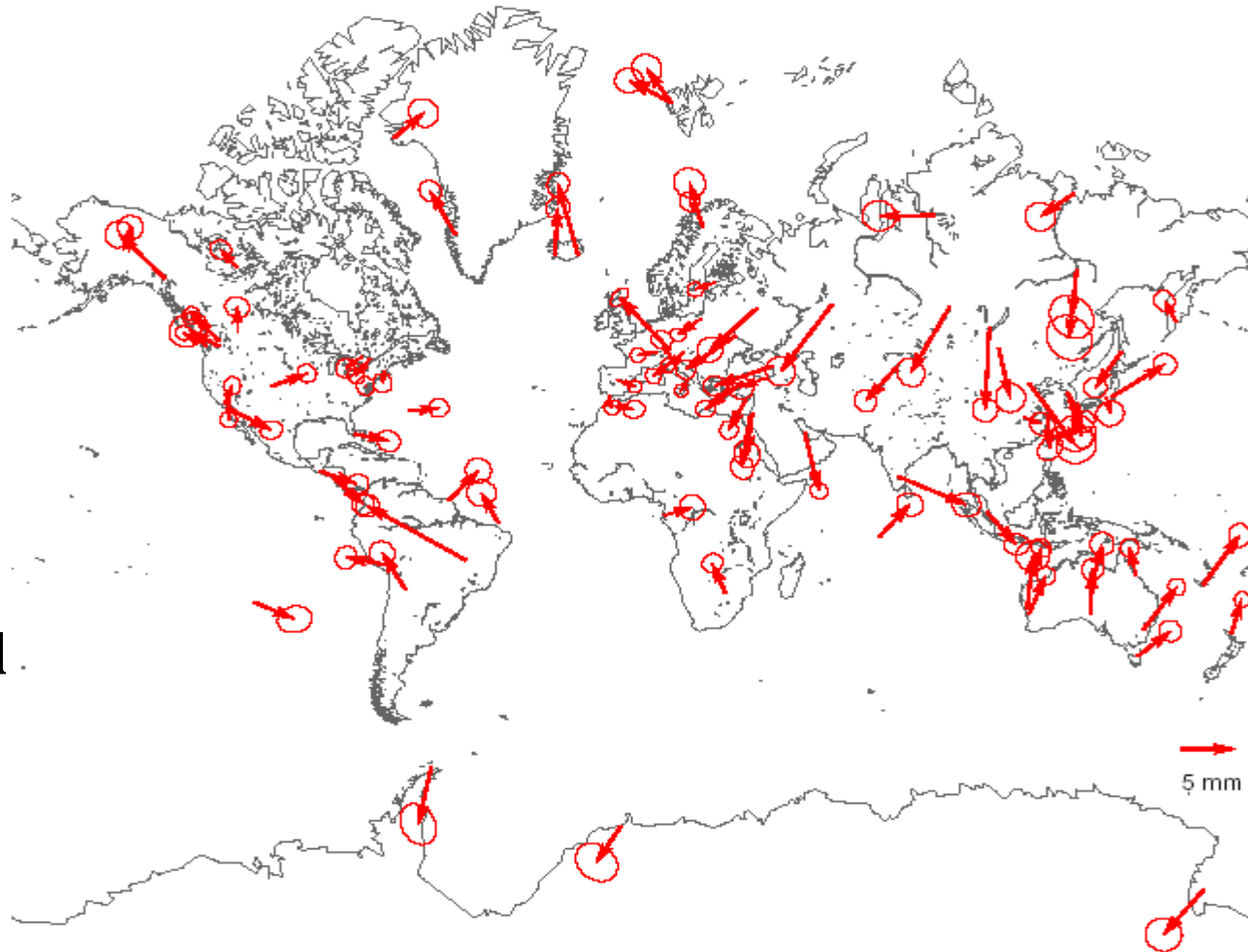
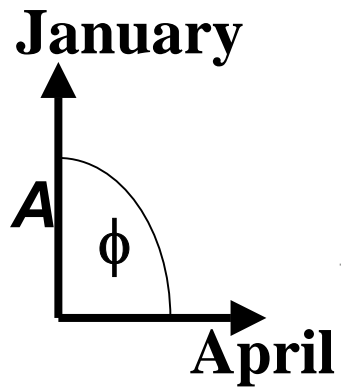
BAHR

DRAO

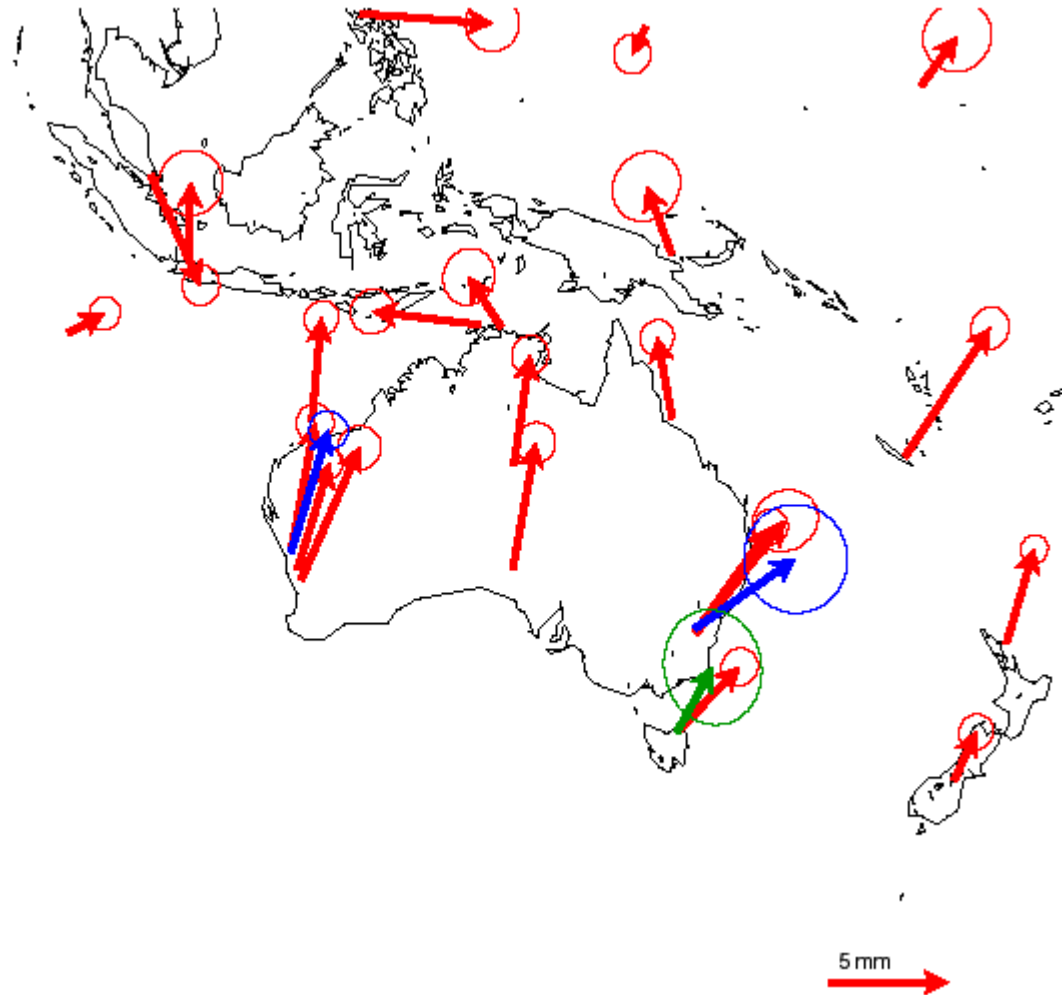
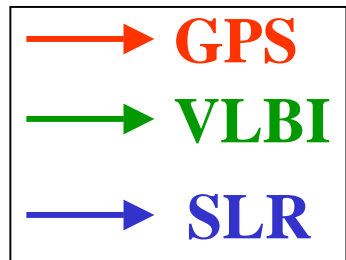
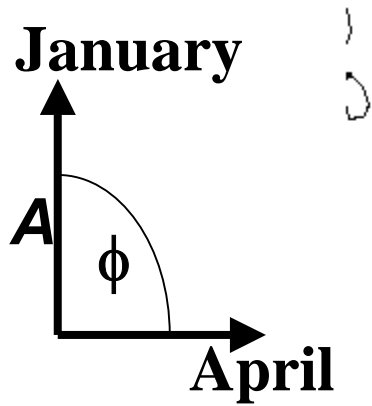
IRKT

Annual Amplitudes & Phase of GPS/IGS station height components

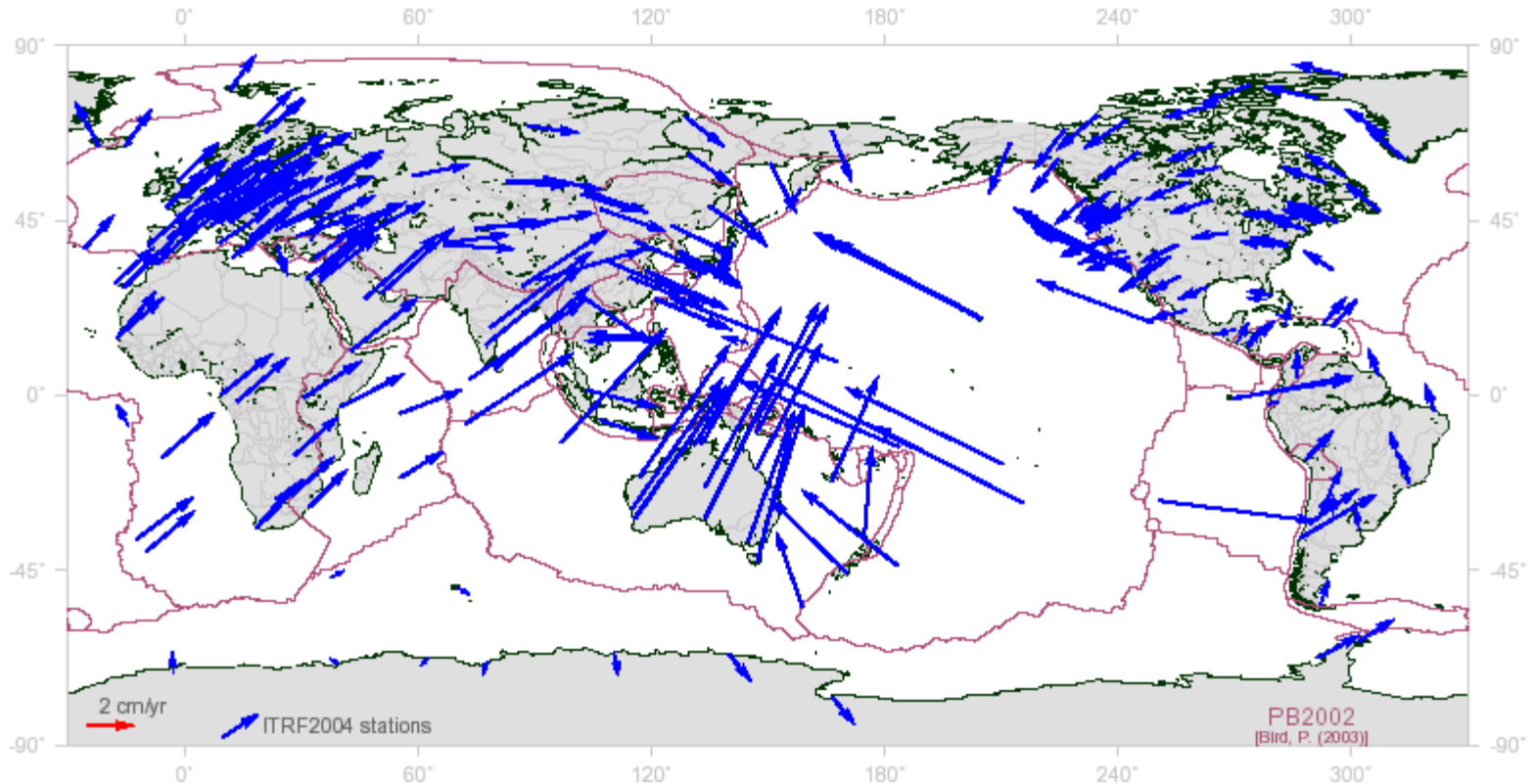
$$Dh = A \cdot \cos(2\pi f(t - t_0) - \phi)$$



Case of Australia



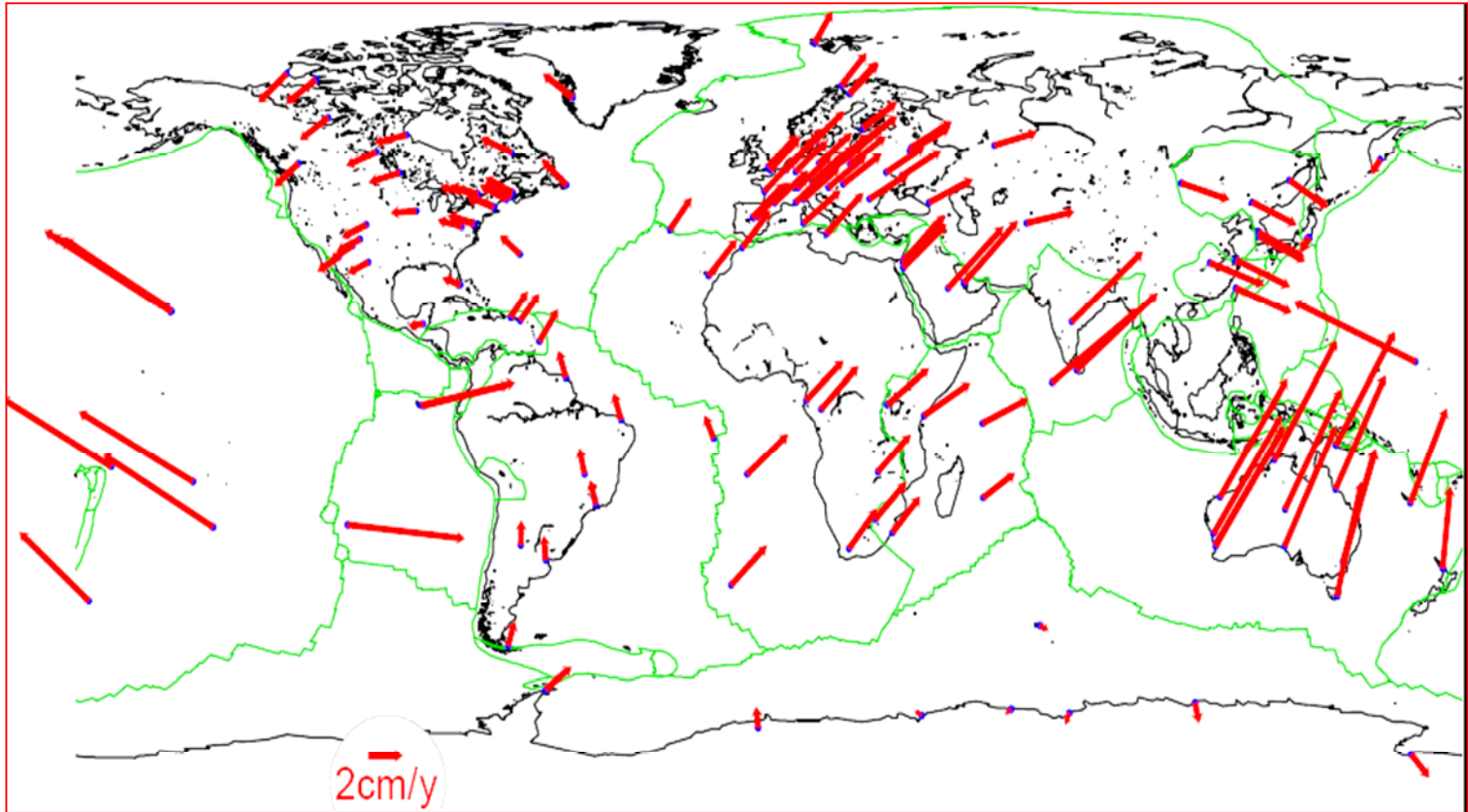
ITRF2005 and Plate motion: Horizontal Site velocities with $\sigma < 3\text{mm/y}$



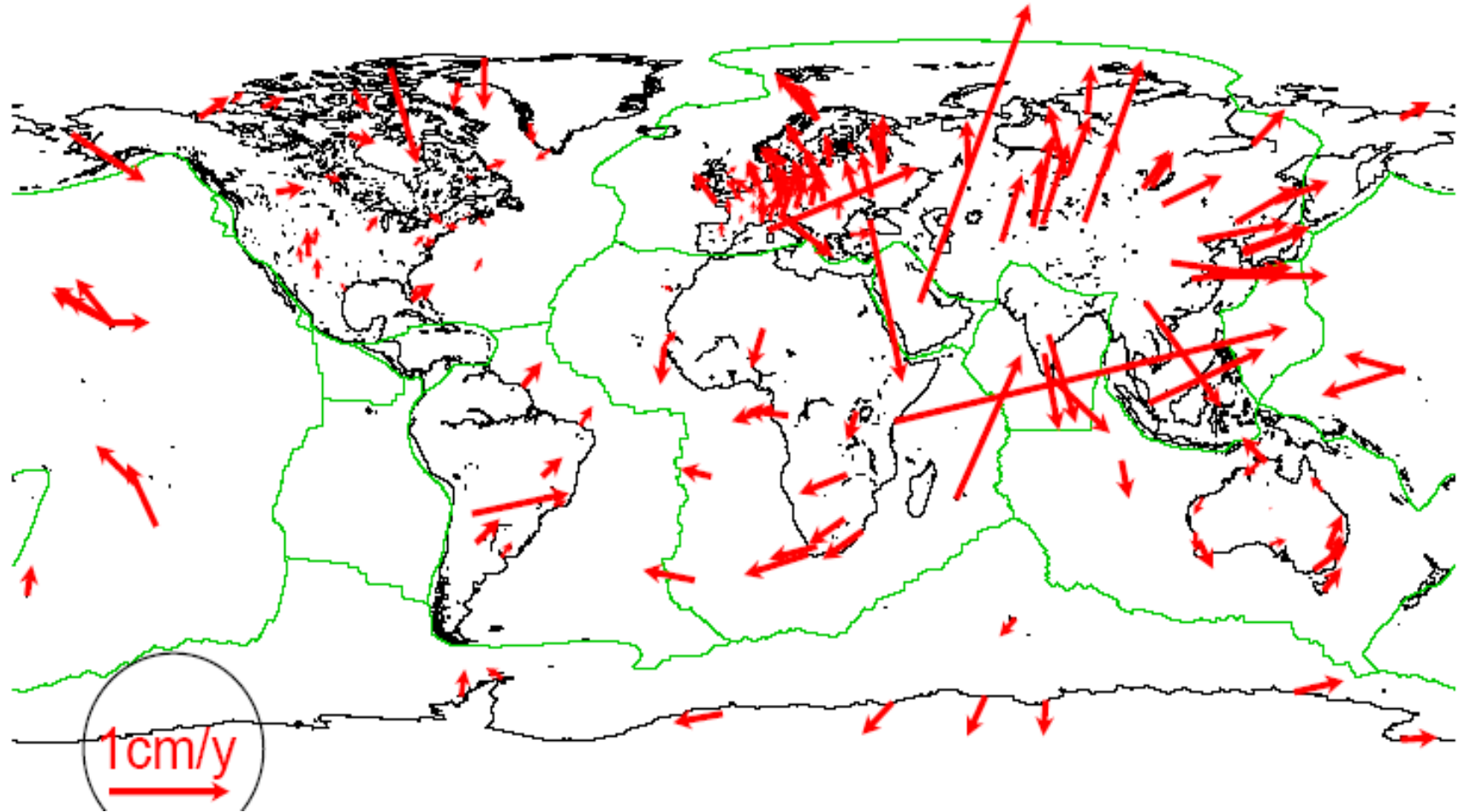
Selected sites for plate angular velocities estimation

Using PB 2002 Plate boundaries (Bird, 2003)

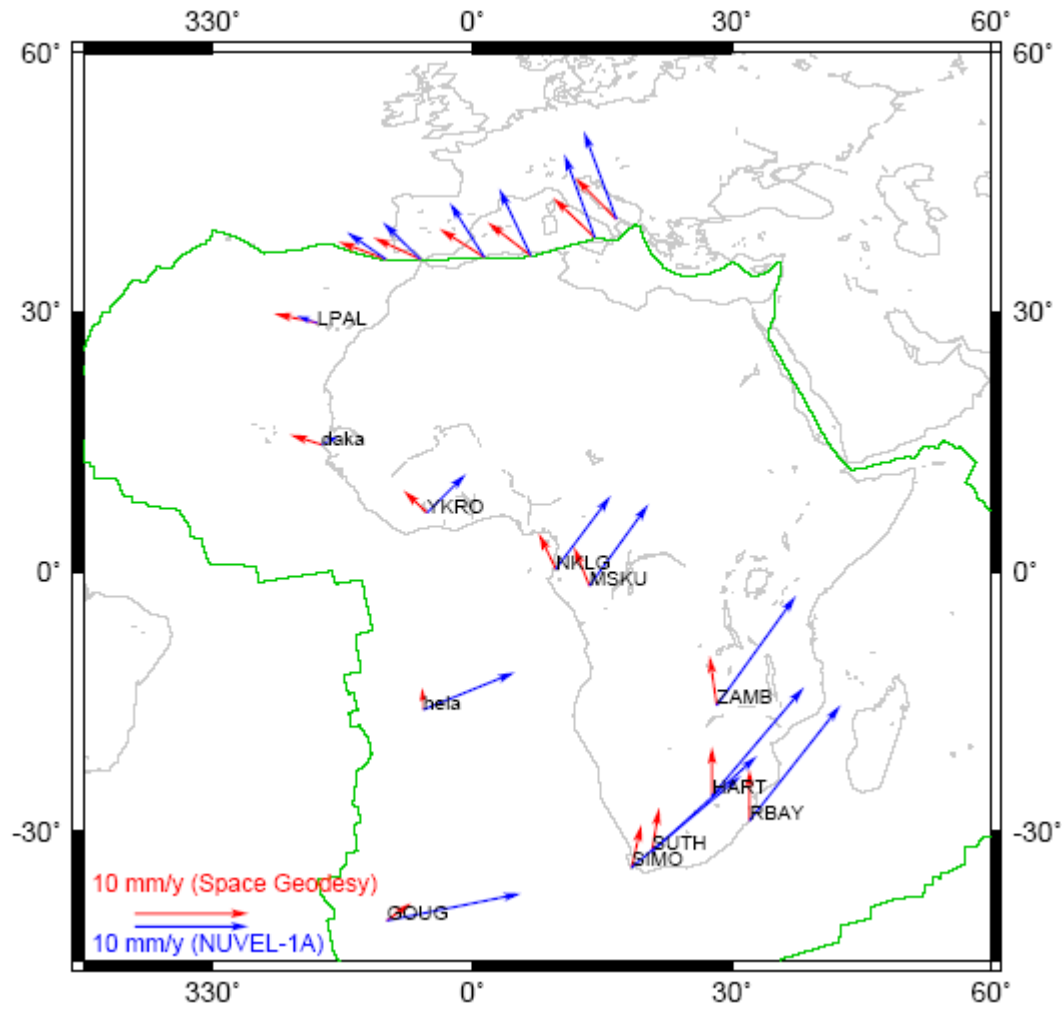
Pacific
Africa
Amur
Antarctica
Arabia
Australia
Caribbean
Eurasia
India
North America
Nazca
Okhotsk
South America
Somalia
Yangtze



Differences ITRF2005 – NNR-NUVEL-1A



AFRC (NUBI) – EURA Motion



Nubia-Eurasia velocity ~50% slower than NUVEL1-A prediction

Transformation Parameters From ITRF2005 to ITRF2000

	TX mm mm/y	TY mm mm/y	TZ mm mm/y	Scale ppb ppb/y
Offset At 2000.0	0.1	-0.8	-5.8	0.40
Drift	-0.2	0.1	-1.8	0.08

ITRF2005

Accuracy of the datum definition

	at epoch 2000.0 (mm)	Rate mm/yr
Origin	5	2
Scale	6.3	0.6
NNR		2

Limitations & Improvements (1/2)

- **Poor SLR & VLBI networks and their co-location**
- **Improve analysis**
 - Systematic errors
 - Include more satellites for SLR ? (see DORIS experience)
 - GM, Satellite CoMs ?
 - Correction models consistency
 - Troposphere
 - Relativity
 - Others
 - More TRF VLBI sessions
 - Process ALL SLR data
- **Improve GPS equipments : discontinuity problem, antenna settings,... !**
- **Improve DORIS scale and Z-component (how ?)**

Limitations & Improvements (2/2)

- **Improve Co-locations :**
 - Re-Survey dubious Co-location sites (International effort needed)
 - **Re-compute all old ties ==> Full SINEX files**
 - **More Co-locations with better distribution : SLR & VLBI !!!**
- **Monitor the ITRF frame parameters (Scale & Origin)**
 - Regular time series analysis
 - Need IAG services commitment to continue providing weekly (daily) solutions
- **Monitor ITRF/EOPs consistency on a regular basis**
 - Coordination between ITRF and EOP PCs



ETRS89 Definition

- **Coincides with ITRS at epoch 1989.0:**
 - **Definition at a reference epoch (1989.0)**
 - **The 7 parameters between ITRS and ETRS89 are zero at 1989.0**
- **Fixed to the stable part of the Eurasian plate**
 - **Co-moving with the plate: law of time evolution**
 - **Time derivatives of the transformation parameters are zero except the 3 rotation rates**

ETRS89 Realization

- Expression in ITRF_{YY} at central epoch (t_c) of the implied observations
- Expression in ETRS89 using 14 transformation parameters some of them are zeros

Positions

$$X^E(t_c) = X_{YY}^I(t_c) + T_{YY} + \begin{pmatrix} 0 & -\dot{R}_{3YY} & \dot{R}_{2YY} \\ \dot{R}_{3YY} & 0 & -\dot{R}_{1YY} \\ -\dot{R}_{2YY} & \dot{R}_{1YY} & 0 \end{pmatrix} \times X_{YY}^I(t_c) \cdot (t_c - 1989.0)$$

Velocities

$$\begin{pmatrix} \dot{X}_{YY}^E \\ \dot{Y}_{YY}^E \\ \dot{Z}_{YY}^E \end{pmatrix} = \begin{pmatrix} \dot{X}_{YY}^I \\ \dot{Y}_{YY}^I \\ \dot{Z}_{YY}^I \end{pmatrix} + \begin{pmatrix} 0 & -\dot{R}_{3YY} & \dot{R}_{2YY} \\ \dot{R}_{3YY} & 0 & -\dot{R}_{1YY} \\ -\dot{R}_{2YY} & \dot{R}_{1YY} & 0 \end{pmatrix} \times \begin{pmatrix} X_{YY}^I \\ Y_{YY}^I \\ Z_{YY}^I \end{pmatrix}$$

ITRFyy Eurasia Rotation Poles

Table 4: Estimation of \dot{R}_{YY}

<i>YY</i>	<i>R1</i> mas/y	<i>R2</i> mas/y	<i>R3</i> mas/y
89	0.11	0.57	-0.71
90	0.11	0.57	-0.71
91	0.21	0.52	-0.68
92	0.21	0.52	-0.68
93	0.32	0.78	-0.67
94	0.20	0.50	-0.65
96	0.20	0.50	-0.65
97	0.20	0.50	-0.65
00	0.081	0.490	-0.792
	± 0.021	± 0.008	± 0.026
05	0.054	0.518	-0.781
	± 0.009	± 0.006	± 0.011

How to realise the ETRS89 ?

- **ITRF_{yy} ==> ETRF_{yy}**

Straightforward: clear transformation formula

- **GPS campaign, weekly solution, others...**

All the problem is how to express first the solution in the ITRF ?

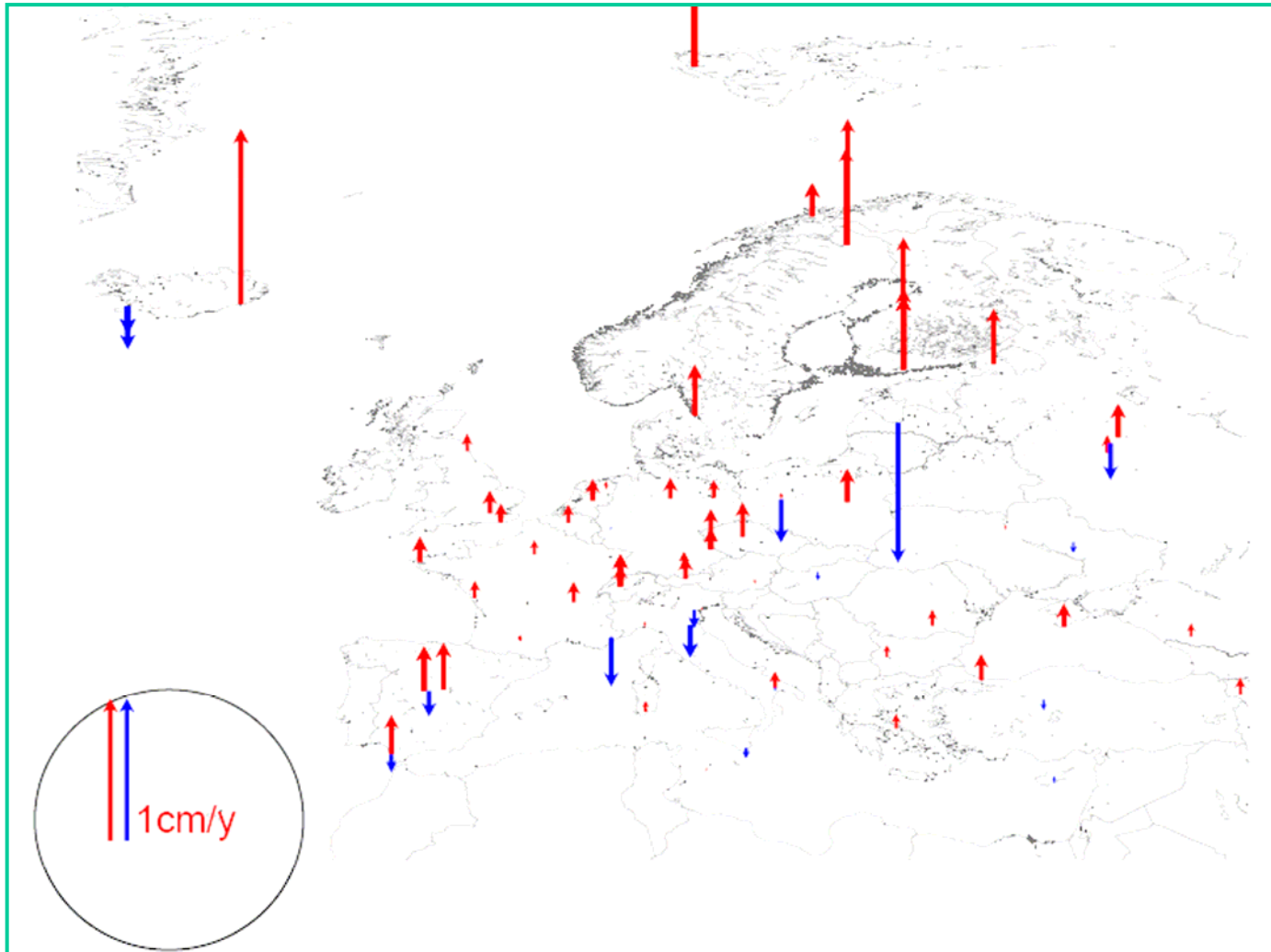
1. **Fixing (constraining) some points to ITRF values**
2. **Using 7-parameter transformation**
3. **Using Minimum constraint approach**

In all cases a reference set of stations is needed

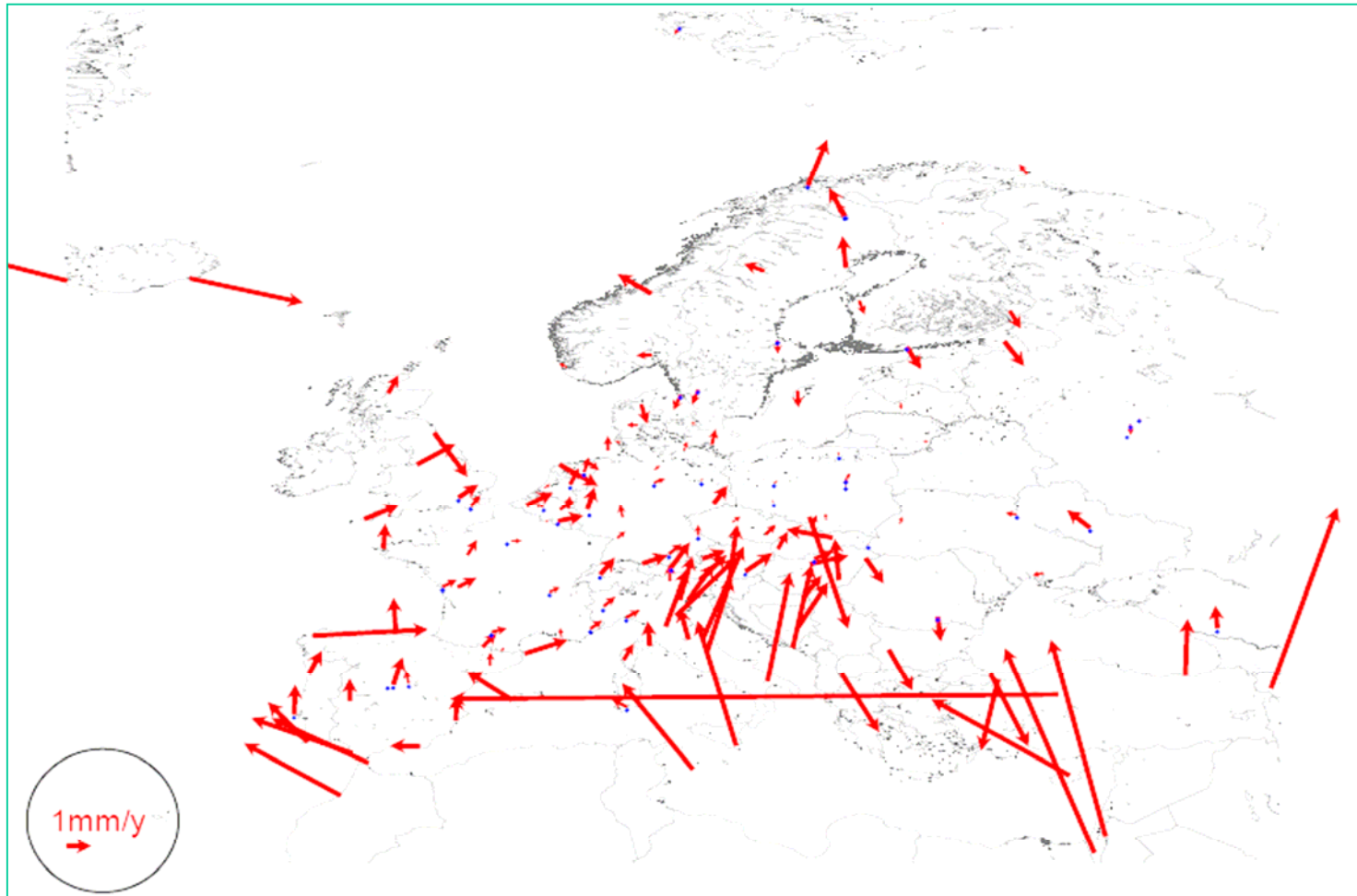
ETRF2005 Horizontal Velocities



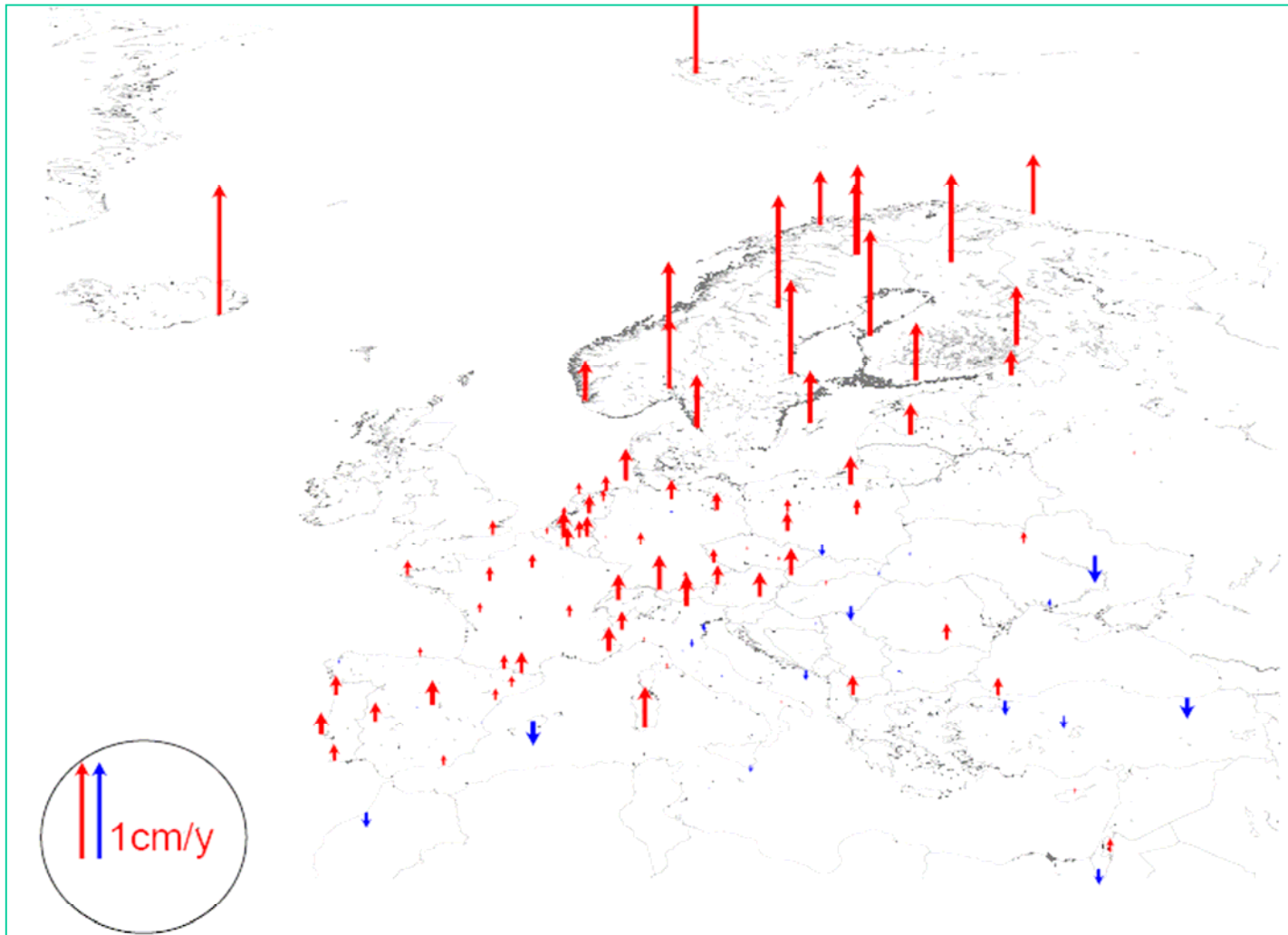
ETRF2005 Vertical Velocities



EPN (ETRF2005) Horizontal Velocities



EPN (ETRF2005) Vertical Velocities



Concluding Remarks

- **Continuous improvement of the ITRF solutions**
- **Time series analysis is critical for ITRF implementation**
- **The ETRS89 is fully linked to ITRS with clear transformation formula**