Conceptual Neighborhoods of Topological Relations Between Lines

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Abstract

Conceptual neighborhood graphs form the foundation for qualitative spatial-relation reasoning as they capture the relations' similarity. This paper derives the graphs for the thirty-three topological relations between two crisp, undirected lines and for the seventy-seven topological relations between two lines with uncertain boundaries. The analysis of the graphs shows that the normalized node degrees increases, from the crisp to the broad-boundary lines, roughly at the same degree as it increases for crisp lines that are transformed from R^1 into R^2.

1. Introduction

Spatial databases need semantically rich geometric data types to describe appropriately spatial configurations. The definitions of such data types include the specification of their data structures, the identification of operations on instances of that type, as well as the identification of the *spatial relations* between such instances. Often these spatial relations are qualitative in nature, abstracting away quantitative details. Topological relations have been the predominantly studied field of such qualitative spatial relations with the identification of binary relations between regions (Egenhofer and Franzosa 1991), lines, and points (Egenhofer and Herring 1991).

The elements of such a set of possible relations are essentially on a *nominal* scale (Stevens 1946), which enables the distinction when two relations are the same or different, but the categorization of relations *per se* yields no further information about the relationships *among* non-equal relations. Such relationships, however, are germane for deciding about higher-level concepts about the spatial relations, such as order, partial order, or similarity. To relate qualitative relations one typically constructs the relations' *conceptual neighborhood graph* (Freksa 1992), which captures explicitly all those pairs of relations that are most similar. Such an organization of a set of relations brings these relations onto a level where more than nominal comparisons can be made, enabling richer analyses.

Nodes in a conceptual neighborhood graph represent spatial relations, while edges are created to connect the relations with the least differences (Bruns and Egenhofer 1996). Since some relations are closer to each other than others, the differences among the relations offer an opportunity to determine the relations' similarities. Paths in the graph refer to sequences of different spatial relations as they result from continuous deformations of the objects. The conceptual neighborhood graph of a set of relations provides a rationale for answering three types of questions:

- Given two relations find the intermediate relations and possible alternative paths, if they exist.
- Given a relation and a particular change process (such as a translation, a rotation, or a scaling), determine set of next possible relations.
- Given two consecutive relations, determine the possible change processes that were involved.

This paper studies the conceptual neighborhood graph of topological relations between *undirected lines*. This graph has been a missing piece in the puzzle of conceptual neighborhood graphs. We pursue a similar approach as for the topological relations between two regions, exploring the graph for crisp lines as well as for lines with broad boundaries. The resulting graphs form a rationale for qualitative similarity reasoning and may also serve in the future as a framework for identifying the semantics of natural-language relations, similar to earlier approaches for relations between lines and regions (Mark and Egenhofer 1994; Kurata and Egenhofer 2007).

The remainder of this paper is structured as follows: Section 2 reviews work on conceptual neighborhood graphs, particularly those for intervals, regions, and directed lines. Sections 3 and 4 derive the conceptual neighborhood graphs for the thirty-three topological relations between two simple, undirected lines and the seventy-seven topological relations between two lines with broad boundaries. Section 5 analyzes these graphs and compares them with the graphs for intervals and regions. Section 6 offers conclusions and discusses items for future work.

2. Conceptual Neighborhood Graphs

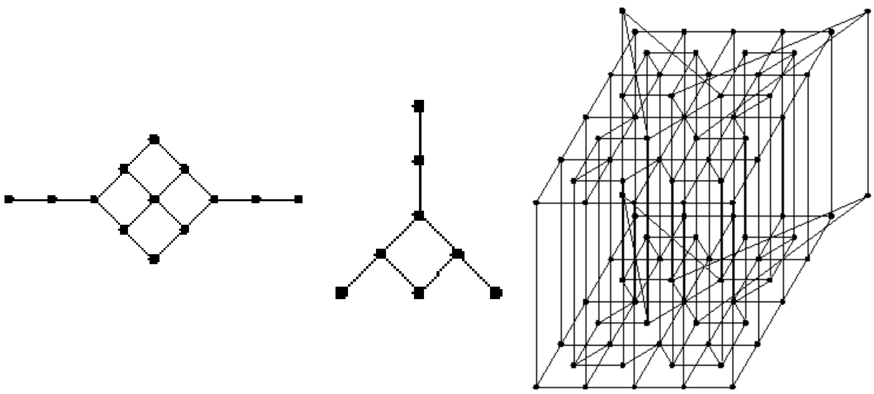
The earliest developments of conceptual neighborhood graphs applied to binary relations between intervals in R^1 (Freksa 1992) and to binary topological relations between regions in R^2 (Egenhofer and Al-Taha 1992). For Allen's (1983) thirteen interval relations a type of similarity is established by moving one of the two ends of an interval while keeping the other end fixed. All possible transitions of this kind are then captured in the conceptual neighborhood graph (Figure 1a). Other types of deformation, such as moving both ends at the same time, leads to somewhat different links, although the overall structure of the graph is preserved.

For topological region-region relations the conceptual neighborhood graph (Figure 1b) was derived from the relations' 9-intersection matrices, considering those pairs of relations as neighbors that feature the least non-zero difference in their matrix elements. In analogy to the interval graph, different types of continuous deformations of the related objects may lead to slightly different graphs, adding occasionally additional links between some nodes. Overall, however, the general framework of the conceptual neighborhood graph remains the same.

Such conceptual neighborhood graphs have been developed for most every set of spatial relations studies, including relations between two cyclic intervals (Hornsby et al. 1999) and between an interval and an interval with a gap (Egenhofer 2007), topological relations between regions on the sphere (Egenhofer 2005) and in Z^2 (Egenhofer and Sharma 1993), for topological relations between regions and lines (Egenhofer and Mark 1995), for topological relations between minimum bounding rectangles (Papadias et al. 1995), convex hulls (Clementini and Di Felice 1997), regions with broad boundaries (Clementini and

Di Felice 1996; Cohn and Gotts 1996), and for the orientation of two lines in the plane (Schlieder 1995).

Most relevant for the development of the conceptual neighborhood of line-line relations is, however, the neighborhood graph for topological relations between two directed lines (Kurata and Egenhofer 2006), which features two parallel layers for relations, one for relations whose lines' interiors do not intersect and another for relations whose interiors do intersect (Figure 1c). In this depiction the nodes along the top and right fringes are repetitions of the nodes along the bottom and the left; therefore, a less redundant conceptual neighborhood graph that shows a single node for each relation warps around the surfaces of two tori, one inside the other, with spike-like connections between the tori, connecting corresponding relations that differ only by their interior-interior intersections.



(a) (b) (c)

Fig. 1. The conceptual neighborhood graphs of (a) the thirteen interval relations in R^1, (b) the eight topological relations between two regions in R^2, and (c) the sixty-eight topological relations between two directed lines in R^2.

3. Conceptual Neighborhood Graph For Topological Relations Between Two Undirected Lines

The 9-intersection distinguishes 33 different topological relations (Figure 2) between two simple, undirected lines (Egenhofer 1993). Such simple lines have exactly two distinct end nodes and feature no self-intersections, bifurcations, or loops. A major difficulty in the development of these relations' conceptual neighborhood graph is the lack of a clear specification of what establishes similarity between two pairs of topological line-line relations. Since these lines are one-dimensional features that are embedded in a two-dimensional space, they have a higher degree of freedom than two lines in R^1 or two regions in R^2, for which continuous deformations, which establish the rationale for neighborhood, are much more constrained and can be carried out in a much more controlled fashion. For example, in R^2 with a single movement two lines that are *equal* (LL 22 in Figure 2) can be transformed so that they are *disjoint* (LL 1). Such an atomic change of relations would be impossible with a continuous transformation for two lines in R^1 (Figure 2a), because in order to become disjoint the lines would need to migrate through intermediate several steps where, at one point, they *overlap* and later *meet*. The analog holds for the transitions of two regions from being *equal* to *disjoint* in R^2 (Figure 1b).

It is this higher degree of freedom that requires a more lenient use of the typical method that has been used to determine the similarity of topological relations, namely to count the differences in all pairs of the relations' 9-intersection matrices and to consider, for each relation, those as neighbors that feature the least differences (Egenhofer and Al-Taha 1992). While this rationale is still valid to determine prime candidates for conceptual neighbors, the single difference count in the matrices is too restrictive at times. For instance, the transition from LL 15 to LL 24 can be accomplished by an atomic deformation, pulling the end of the line from the other line's interior to its boundary. This change, however, implies a difference of two in their matrices, because the moved line's boundary moves not only out of the interior (one change) but also into to other line's boundary (a second change). Since both LL 15 and LL 24 have other neighbors that differ by only one count, the least difference in matrix elements does not lead appropriate to identifying neighboring relations.

Therefore, we start deriving the lines' conceptual neighborhood graph from the graph of directed lines. Both sets of relations have relation pairs that differ only by their interior intersections, which creates for the line-line relations a similar dichotomy as for the relations between two directed lines. Another tempting inference cannot be made, however. Although each line can be refined with two orientations—one from a start node to the end node, and the other in the reverse direction—this distinction does not double the number of relations, because in a number of cases the distinction of two orientations is immaterial for the directed lines' topological relation, while in others it means that for each line-line relation there are four refinements due to the lines' orientations.

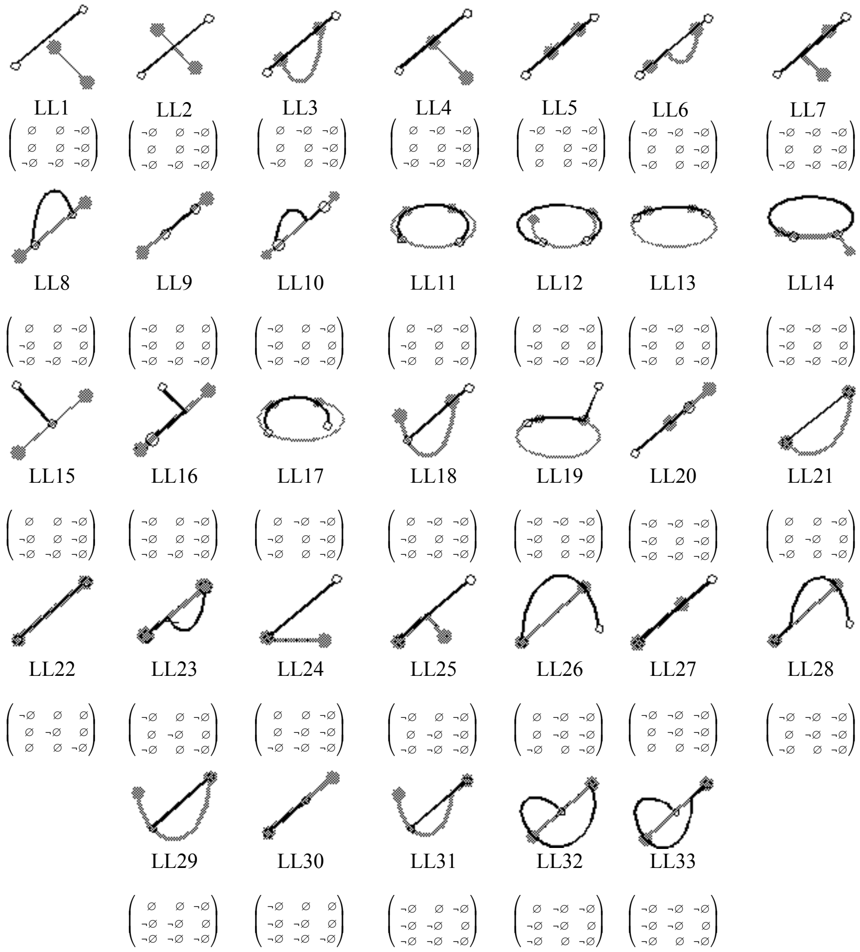


Fig. 2. The thirty-three topological relations between two undirected lines identified by the 9-intersection (Egenhofer 1995).

Within a layer of the directed-line neighborhoods, this impact of the orientation on the line can be observed directly (Figure 3a). The graph's horizontal and vertical axes partition it into four quadrants, each of which captures for the same line-line relation a different aspect of the directed lines' orientations. At the intersection of the two axes is a relation for which the lines' orientations have no impact. If for all relations the orientation of one directed line is ignored, then those relations merge with the mirror images in the graph, either along the horizontal or the vertical axis. If subsequently the other line's orientation is ignored as well, then another merger occurs, this time of the remaining relations along the second axis, leaving as the relations between two undirected lines those that fall into a single quadrant (plus those that coincide with the mirror axes). Therefore, the framework of the conceptual neighborhood graph for the relations between two undirected lines resorts to thirteen relations located in one of the four quadrants (Figure 3b). The parallelism of line-line relations with empty and non-empty interior-interior intersections yields two parallel layers, in which each corresponding pair of line-line relations is connected (Figure 3c).

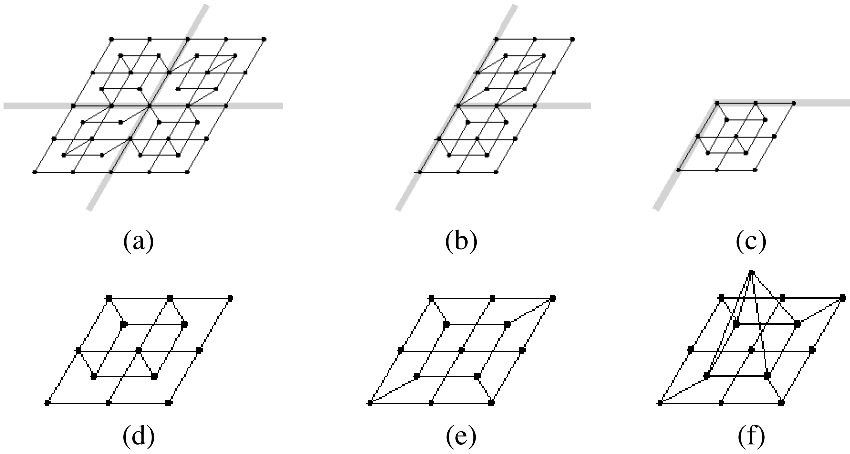


Fig. 3. Deriving a layer in the conceptual neighborhood graph of the line-line relations from the graph for directed lines: (a) the mirror axes in the directed lines' graph; (b) the graph after ignoring the orientation for one of the two lines; (c) the graph after ignoring the orientation of the second line; (d) the derived framework for the conceptual neighborhood graph of the line-line relations; (e) the same framework after adjusting for differences due to the consideration of the lines' exteriors; and (f) the pyramid formed to connect the relation with coinciding boundaries to the layer.

The conceptual neighborhood of the directed-lines' relations was derived from matrix differences of the *head-body-tail intersection* (Kurata and Egenhofer), however. Unlike the 9-intersection it ignores intersections with exteriors, therefore, some of the connections within a quadrant need to be adjusted for the 9-intersection-based line-line relations (Figures 3d and 3e). For the line-line relations' conceptual neighborhood graph, all connections across the two layers now reflect a single difference in the corresponding 9-intersection matrices.

Special attention needs to be paid to two irregularities, however. First are the relations where the two lines' nodes coincide are not integrated into the regularity of the two layers, but—for directed lines—yield two spikes that come out of the layer. When ignoring the lines' orientations, these spikes are merged into a single relation. From this relation, the least matrix difference to any of the relations in the layers is two, not one as among the other relations in a layers, which provides further evidence for that relation's location in the graph. There are four such connections of length 2 in a layer, distributed equally around the central node, which yields a pyramid that emerges from a layer (Figure 3f).

The second exception is the remaining set of five relations (i.e., 33 line-line relations minus 2 layers, each at 13 relations, minus 2 times one spiked relations), which do not yet fit into the overall framework. These five relations include equal (LL 22), the direction-independent versions of starts and finishes (LL 27 and LL 30), and the true subset and its converse relation (LL 5 and LL 9). All five have a non-empty interior-interior intersection, which puts them to the layer with the non-empty interior-interior intersections. Relations LL 5, LL 9, LL 27, and LL 30 each have a neighbor with a one-unit difference to the layer with the non-empty interior-interior intersections. Also LL 27 and LL 30 are within one unit from LL 5 and LL 9, respectively.

These considerations lead to a conceptual neighborhood graph (Figure 4) that strongly resembles the graph of the directed-line relations (Figure 1c). It features again two connected layers, each with a spiked node. In addition, there is a reduced top layer connected to the relations with non-empty interior-interior intersections, including a spiked node. The line-line graph, however, lacks the repetition of relations along the fringes so that no warping into a higher space is suggested.

This conceptual neighborhood graph of the topological line-line relations exhibits some of the properties that have been found with other relations' graphs that were derived from the matrix differences of their 9-intersections.

- Pairs of relations that differ by one entry in their relation matrices form the connections between the extreme landmarks of the conceptual neighbors.
- A one-unit difference is not necessarily possible for all relations within a connected graph. In some cases (e.g., from LL 21 to LL 24), the smallest difference between two neighbors may be two units in the matrix differences. It confirms the insight first gained with the conceptual neighborhood graph for region-region relations where the matrices of the neighbors *equal* and *covers*, as well as *equal* and *coveredBy*, are three units apart (Egenhofer and Al-Taha).

- The least number of differences is not necessarily symmetric. For instance, from LL 23 the nearest neighbors are LL 25 and LL 21, because their matrices differ from LL 23 by one unit. On the other hand, LL 22 differs from LL 23 by two matrix elements so it would not have the least number of differences from LL 23 to be considered a neighbor. Reversely, however, the two-unit difference between LL 22 and LL 23 is the least difference for any pair involving LL 22 so that this pair forms a conceptual neighbor. Again this property is occurs with the region-region graph as well, where the nearest neighbor for *meet* is *disjoint* (distance 1), while *overlap* (with distance 3) would not qualify as a neighbor. In the reverse direction, however, *overlap*'s nearest neighbors have all distances of three, among them *meet*, so that *meet* and *overlap* are neighbors in the graph.
- If only symmetric least differences are accounted for neighbors, then the conceptual neighborhood graph would be disconnected, preventing comparisons across the separations. For example, relations LL 22 would be isolated in the graph since all its neighbors—LL 23, LL 27, and LL 30—themselves have neighbors with smaller matrix differences.

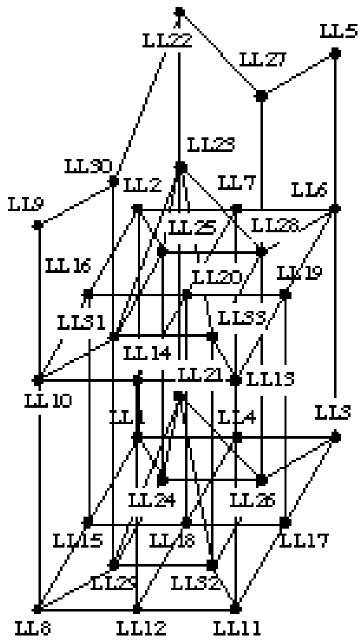


Fig. 4. The conceptual neighborhood graph of the thirty-three topological relations between two lines.

4. Conceptual Neighborhood Graph For Topological Relations Between Two Broad-Boundary Lines

The thirty-three topological line-line relations rely on a crisp representation of the lines. Often, however, this requirement cannot be guaranteed so that more ambiguous line representations should be considered when analyzing their topological relations. This corresponds to the transition of region-region relations from crisp regions to regions with broad boundaries (Clementini and Di Felice 1996; Cohn and Gotts 1996).

Several models are in practice for uncertain lines. *Broad lines* attach uncertainty uniformally around a line (Chrisman 1982), *broad-boundary lines* (Clementini and Di Felice 1997) consider uncertainty only for a line's end points, and *uncertain lines* (Clementini 2005) propagate uncertainly from the boundaries to the interior (Dutton 1992). While the geometric differences of these models may be minor with respect to individual lines, their impact on the possible topological relations is significant: broad lines give rise to five different topological relations (Reis et al. 2006), uncertain lines yield 146 topological relations (Clementini 2005), and broad-boundary lines feature 77 different relations (Reis et al. 2006). We compare the 33 crisp line-line relations with the 77 broad-boundary lines, because their numbers of relations come closest to each other. The broad-boundary relations have been formally derived with the 9-intersection and geometrically verified, which is a process that confirms their existence (Figure 5).

The model for broad-line relations maps onto a subset of the eight region-region relations (Egenhofer and Franzosa 1991) that is obtained by merging partial and complete containments, both with respect to interiors and exteriors, so that the differences between the three pairs of relations of *disjoint–meet*, *covers–contains*, and *coveredBy–inside* are ignored. Therefore, the conceptual neighborhoods can be derived directly from conceptual neighborhood graph of the region-region relations (Egenhofer and Al-Taha 1992). The 146 relations between uncertain lines have been determined computationally, but lack to date a geometric verification.

To determine their conceptual neighborhoods, we employ the snapshot model (Egenhofer and Al-Taha 1992) as well as the smooth-transition model (Egenhofer and Mark 1994). Both methods provided the same pairs of neighbors for the seventy-seven relations. The conceptual neighborhood graph for the broad-boundary relations features two subgraphs—one capturing the 25 relations with empty interior-interior intersections (Figure 6) and another one the 52 relations with non-empty interior-interior intersections (Figure 7).

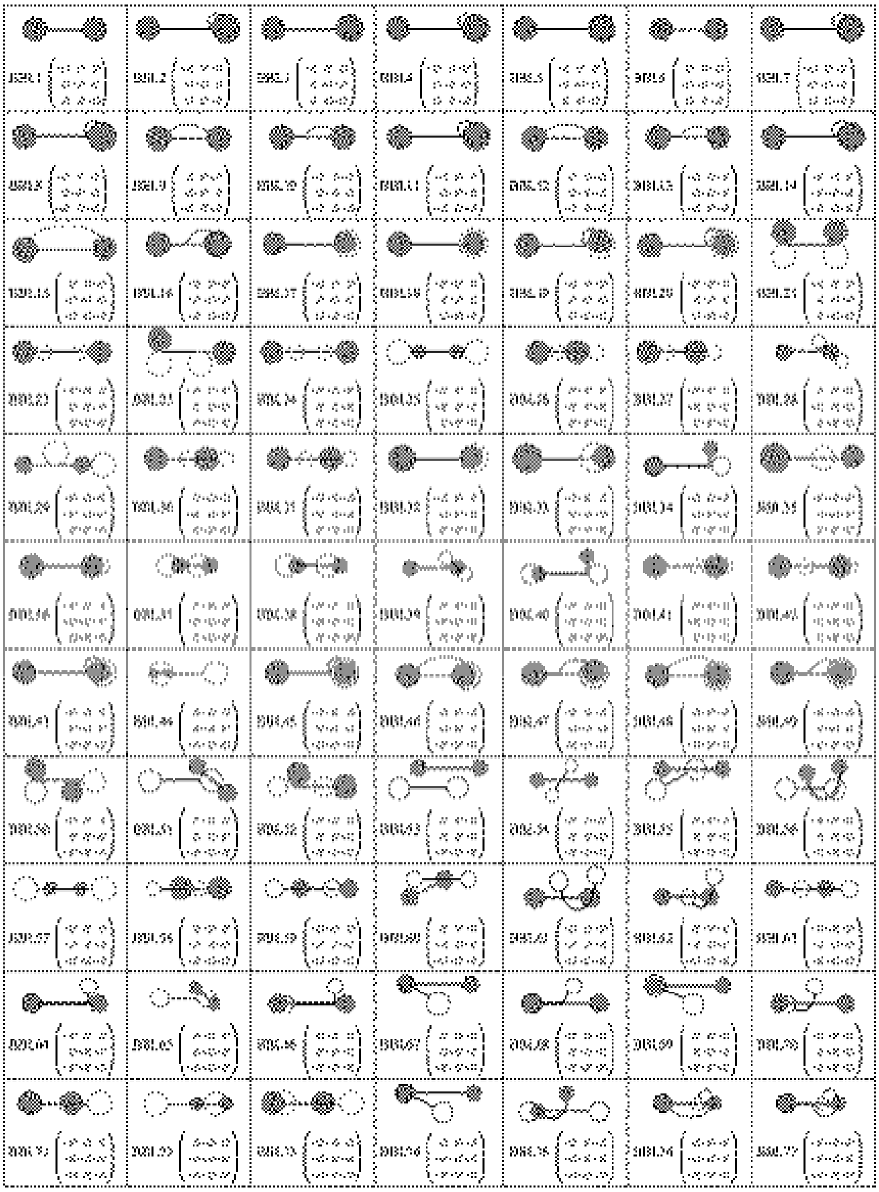


Fig. 5. Seventy-seven topological relations between two broad-boundary lines derived from the 9-intersection (Reis et al. 2006).

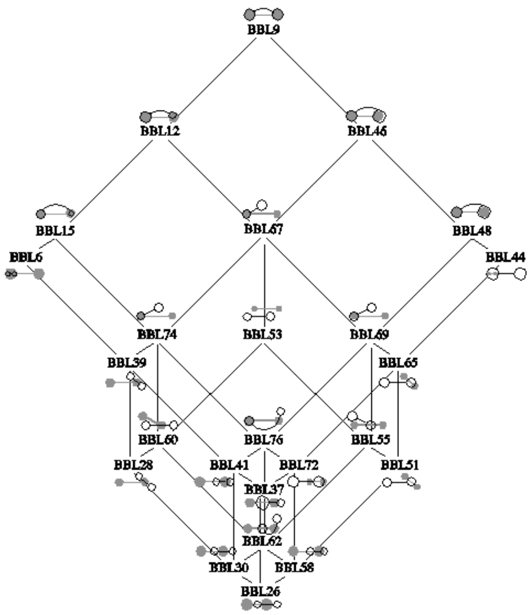


Fig. 6. Conceptual neighborhood graph of broad-boundary lines: layer of relations with empty interior-interior intersections.

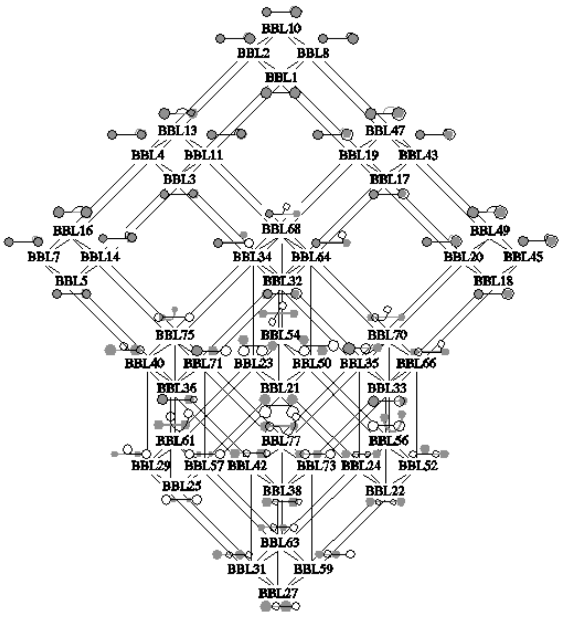


Fig. 7. Conceptual neighborhood graph of broad-boundary lines: layer with non-empty interior-interior intersections.

Both subgraphs have the same overall pattern with two rhombi—a larger one and a smaller one—that are interleaved but do not intersect. The graphs emphasize that 17 of the 77 relations are symmetric (they are all located along the vertical centerlines) and that the remaining 60 relations are made up of 30 pairs of converse relations. Each pair of converse relations is located as a mirror-image along the graph's vertical center line. The 9-intersection matrices of each converse pair are also mirror images of each other, taken along the main diagonal of the matrices. The graphs' structure also highlights fifteen 4-tuples of relations, which form a rhombus based on the four relations' 9-intersections. Within each rhombus the matrices follow the same pattern (Figure 8), that is, seven intersections are fix, while the interior-exterior and exterior-interior intersections cycle through all possible empty and non-empty combinations.

$$\begin{pmatrix} a & b & \neg\varnothing \\ c & d & e \\ \neg\varnothing & f & h \end{pmatrix}$$

$$\begin{pmatrix} a & b & \neg\varnothing \\ c & d & e \\ \varnothing & f & h \end{pmatrix} \qquad \begin{pmatrix} a & b & \varnothing \\ c & d & e \\ \neg\varnothing & f & h \end{pmatrix}$$

$$\begin{pmatrix} a & b & \varnothing \\ c & d & e \\ \varnothing & f & h \end{pmatrix}$$

Fig. 8. The repeated pattern of the 9-intersection matrices in each rhombus of the conceptual neighborhood graph of the regions between two lines with broad boundaries

Each of the 25 nodes in the empty-interior-interior subgraph (Figure 6) has a corresponding node in the non-empty-interior-interior subgraph (Figure 7). The 9-intersection matrices of these 25 pairs differ by one unit; therefore, each pair is a neighbor as well and their connections (as vertical links) yield a single, connected conceptual neighborhood graph with the familiar 2-layered structure (Figure 9).

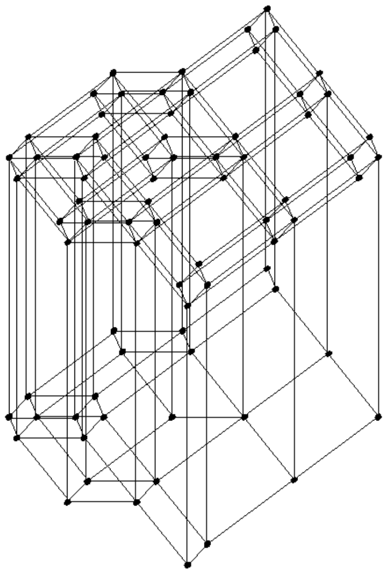


Fig. 9. The conceptual neighborhood graph for the seventy-seven topological relations between two broad-boundary lines.

5. Comparisons of Conceptual Neighborhood Graphs

Beyond the pure visual comparison of these conceptual neighborhood graphs, we offer some quantitative means and apply them to the graphs of two regions (Figure 1b), two lines in R^1 (Figure 1a), two lines in R^2 (Figure 4), and two broad-boundary lines in R^2 (Figure 8).

The *number of nodes* in each graph corresponds to the number of relations for which the graph captures their neighborhood. The *node degree* captures, for each relation, the number of conceptual neighbors. Relations with a node degree of 1 have exactly one conceptual neighbor, relations with a node degree of 2 have two neighbors, and so on. We count in each graph how many relations have a particular node degree. The *node degree sum* is the total node degree of a graph (which is always equal to twice the number of edges in a neighborhood graph). Since graphs with more nodes have a tendency to have a higher node degree, we also normalize the node degree by the number of nodes in a graph, which yields the graph's *normalized node degree*.

Each conceptual neighborhood graph has a lower and upper bound for the node degree. The *lower bound* (Eqn. 1a) is the node degree sum of a linear graph that connects all nodes such that there are two end nodes, each with a node degree of 1, and each of the remaining nodes has a node degree of two. The *upper bound* (Eqn. 1b) is the node degree of a complete graph (i.e., each pair of nodes in connected by exactly one edge).

The *minimality* (Eqn. 1c) and the *saturation* (Eqn. 1d) are two measures that compare a graph's node degree to its lower and upper bound, respectively.

$$lowerBound(G) = 2 * (\#\, nodes(G) - 2) + 2 \tag{1a}$$

$$upperBound(G) = \#\, nodes(G) * (\#\, nodes(G) - 1) \tag{1b}$$

$$minimality(G) = \frac{nodeDegreeSum(G)}{lowerBound(G)} - 1 \tag{1c}$$

$$saturation(G) = \frac{nodeDegreeSum(G)}{upperBound(G)} \tag{1d}$$

Table 1 summarizes the measures for the four conceptual neighborhood graphs.

Table 1. Summary of quantitative comparison of the conceptual neighborhood graphs between two regions in R^2, two lines in R^1, two lines in R^2, and two broad-boundary lines in R^2

	regions in R^2	lines in R^1	lines in R^2	broad-boundary lines in R^2
node cardinality	8	13	33	77
node degree 1	3	2	0	0
node degree 2	2	4	2	0
node degree 3	3	6	3	3
node degree 4	0	1	24	13
node degree 5	0	0	1	31
node degree 6	0	0	1	24
node degree 7	0	0	0	5
node degree 8	0	0	0	1
node degree sum	16	32	120	403
norm. node degree	2.00	2.46	3.64	5.23
lower bound	14	24	64	152
upper bound	56	156	1,056	5,852
minimality	14%	33%	47%	165%
saturation	28.6%	20.5%	11.4%	6.9%

We establish two baselines for assessing the graphs' node degrees: (1) the region-region relations, (2) the line-line relations in R^1, and (3) the line-line relations in R^2 (Table 2). The first case shows that the graphs' normalized node degrees increase from regions to lines. Case 1 and 2 also show that the graphs' normalized node degrees increase progressively more for broad-boundary lines. The increase in the normalized node degree, however, is less than the increase in the number of nodes. Finally the cross-comparison shows that the transition from lines in R^1 to lines in R^2 has roughly the same impact on the graphs' normalized node degrees as the transition from lines to broad-boundary lines (a 48% increase vs. a 44% increase).

Table 2. Comparisons of the increases in number of nodes and normalized node degree for the conceptual neighborhood graphs of two regions in R^2, two lines in R^1, two lines in R^2, and two broad-boundary lines in R^2

	regions in R^2	lines in R^1	lines in R^2	broad-boundary lines in R^2
nodes	100%	+63%	+313%	+863%
norm. node degree	100%	+23%	+82%	+162%
nodes		100%	+153%	+492%
norm. node degree		100%	+48%	+113%
nodes			100%	+133%
norm. node degree			100%	+44%

7. Conclusions and Future Work

The conceptual neighborhood graphs for topological relations between two crisp lines in R^2 between two lines broad-boundary lines in R^2 have a similar structure due to the parallel occurrence of line relations that share an interior-interior intersections and those that do not. Both graphs are not planar, but still show a highly regular structure. The quantitative comparison showed that the increase in the normalized node degree from crisp to broad-boundary lines is roughly at the same degree as the increase from the mapping of lines from R^1 into R^2.

It is worthwhile to analyze in the future whether a similar behavior can be found for the relations of regions and broad-boundary regions. Beyond the node degree, additional measures on the graphs should be considered to analyze the distribution of the maximum path lengths in the graphs, because the maximum path lengths are used as a measure to normalize

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